

DUAL NATURE OF RADIATION AND MATTER

11.1 FREE ELECTRONS IN METALS

1. Why do metals have a large number of free electrons? Define work function of a metal. Give the unit of work function.

Free electrons in metals. In metals, the electrons in the outer shells of the atoms are loosely bound. They move about freely throughout the lattice of positive ions. Such loosely bound electrons are called *free electrons*.

The free electrons, however, remain confined to the conductor and cannot leave its surface at ordinary temperature and under moderate electric fields. The moment an electron comes out of a metal surface with its negative charge ($-e$), the metal surface acquires an equal positive charge ($+e$) and pulls it back. There is thus a *potential barrier* at the metal surface which the free electrons have to overcome in order to just escape from the metal surface.

The minimum amount of energy required by an electron to just escape from the metal surface is called **work function** of the metal.

The work function depends on (i) the nature of the metal and (ii) the conditions of its surface. It is generally denoted by W_0 (or ϕ_0) and measured in electron volt (eV).

Electron volt. One electron volt is the kinetic energy gained by an electron when it is accelerated through a potential difference of 1 volt.

Energy gained by electron

$$= \text{Work done by electric field} = qV$$

$$\therefore 1 \text{ eV} = 1.602 \times 10^{-19} \text{ C} \times 1 \text{ V}$$

or $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$

Electron volt is a commonly used unit of energy in atomic and nuclear physics.

Table 11.1 Work Functions of some Photosensitive Metals

Metal	Work function (eV)	Metal	Work function (eV)
Cs	2.14	Al	4.28
K	2.30	Hg	4.49
Na	2.75	Cu	4.65
Ca	3.20	Ag	4.70
Mo	4.17	Ni	5.15
Pb	4.25	Pt	5.65

It may be noted from the above table that the work function of platinum is the highest ($W_0 = 5.65 \text{ eV}$) while it is lowest for caesium ($W_0 = 2.14 \text{ eV}$).

11.2 ELECTRON EMISSION

2. What is electron emission? Briefly explain different methods of electron emission.

Electron emission. The phenomenon of emission of electrons from a metal surface is called electron emission. For the emission of electrons from the metal surface,

the energy of electrons must be higher than the work function of the metal. For their release from the metal surface, the electrons may be supplied the required amount of energy by any one of the following methods :

1. **Thermionic emission.** When a metal is heated, its free electrons get sufficient thermal energy and they can overcome surface barrier. This method of removal of electrons is called *thermionic emission* and the emitted electrons are called *thermions* or *thermo-electrons*.

2. **Field emission or cold cathode emission.** When a metal surface is subjected to very high electric fields, of the order of 10^3 to 10^8 Vm^{-1} , electrons are emitted from it. This is known as *field* or *cold cathode emission*. This method of electron emission is dangerous and less efficient.

3. **Photoelectric emission.** It is the process in which electrons are emitted from a metal surface when electromagnetic radiations of sufficiently high frequency are incident on it. The emitted electrons are called *photoelectrons* but their rate of emission is very low.

4. **Secondary emission.** When fast moving electrons strike a metal surface, they transfer some of their energy to the free electrons of the metal. As a result, the energy of the free electrons becomes more than the work function of the metal and they get ejected from the metal surface. The emitted electrons are called *secondary electrons* and this method of removal of electrons is called *secondary emission*.

For Your Knowledge

- Free electrons in a metal are free in the sense that they move inside the metal in a constant potential. They are not free to come out of the metal. Additional energy is needed to eject them out from the metal surface.
- All the free electrons in a metal do not have the same energy. At a constant temperature, these electrons have a certain energy distribution.
- Because of the energy distribution of free electrons in a metal, the energy required to eject an electron is different for different electrons. An electron with higher energy requires less additional energy to come out of the metal than that with lower energy. *Work function* is the minimum energy required by an electron to get out of the metal surface.

11.3 PHOTOELECTRIC EFFECT

3. Describe the phenomenon of photoelectric effect. Briefly describe the observations of Hertz, Hallwachs and Lenard in this regard.

Photoelectric effect. When light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface. The phenomenon is called *photoelectric effect*.

Hertz's observations. The phenomenon of photoelectric effect was discovered by *Heinrich Hertz* in 1887. While demonstrating the existence of electromagnetic waves, Hertz found that high voltage sparks passed across the metal electrodes of the detector loop more easily when the cathode was illuminated by ultraviolet light from an arc lamp. The ultraviolet light falling on the metal surface caused the emission of negatively charged particles, which are now known to be electrons, into the surrounding space and hence enhanced the high voltage sparks.

Hallwachs' and Lenard's Observations. During the years 1886 – 1902, *Wilhelm Hallwachs* and *Philipp Lenard* investigated the phenomenon of photoelectric emission in detail.

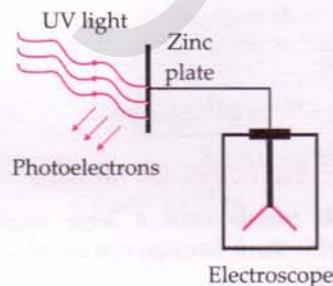


Fig. 11.1 Demonstration of photoelectric effect.

As shown in Fig. 11.1, Hallwachs connected a zinc plate to an electroscope. He allowed ultraviolet light to fall on a zinc plate. He observed that the zinc plate became (i) uncharged if initially negatively charged, (ii) positively charged if initially uncharged and (iii) more positively charged if initially positively charged. From these observations, he concluded that some negatively charged particles were emitted by the zinc plate when exposed to ultraviolet light.

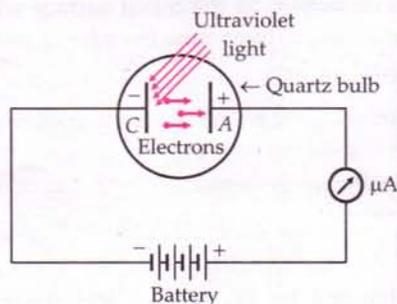


Fig. 11.2 Production of photoelectric current.

A few years later, Lenard observed that when ultraviolet radiations are allowed to fall on the emitter

plate of an evacuated glass tube enclosing two electrodes (cathode C and anode A), a current flows in the circuit. As soon as ultraviolet radiations are stopped, the current also stops. These observations again indicate that ultraviolet radiations incident on the emitter plate C eject out same negatively charged particles from it. These particles are attracted by the collector plate A , setting up a current through the evacuated glass tube. After the discovery of electrons by J.J. Thomson in 1897, it was established that these particles are indeed electrons and were called *photoelectrons*. In 1900, Lenard argued that when ultraviolet light is incident on the emitter plate, it causes the emission of electrons from its surface. These electrons are attracted by the positive collector plate so that the circuit is completed and a current flows. This current was called *photoelectric current*.

Hallwachs and Lenard also observed that when the frequency of the incident light was less than a certain minimum value, called the *threshold frequency*, no photoelectrons were emitted at all.

The phenomenon of emission of electrons from a metal surface, when electromagnetic radiations of sufficiently high frequency are incident on it, is called **photoelectric effect**. The photo (light)-generated electrons are called **photoelectrons**.

Different substances emit photoelectrons only when exposed to radiations of different frequencies. Alkali metals like Li, Na, K, Cs and Rb are *highly photosensitive*. They emit electrons even with visible light. Metals like Zn, Cd, Mg, Al, etc. respond only to ultraviolet light. X-rays can eject electrons even from heavy metals.

11.4 EXPERIMENTAL STUDY OF PHOTOELECTRIC EFFECT

4. Describe an experimental arrangement to study photoelectric effect. Explain the effect of (i) intensity of light on photoelectric current, (ii) potential on photoelectric current and (iii) frequency of incident radiation on stopping potential.

Experimental study of photoelectric effect. An extensive study of photoelectric effect was made by Lenard and R.A. Millikan.

Figure 11.3 shows the experimental arrangement used for the study of photoelectric effect. It consists of an evacuated glass/quartz tube which encloses a photo-sensitive plate C and another metal plate A . A quartz window W is sealed on the glass tube which permits the ultraviolet light to irradiate the plate C . The

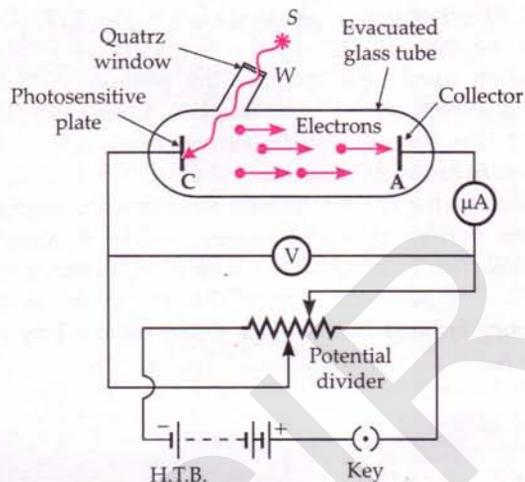


Fig. 11.3 Experimental arrangement to study photoelectric effect.

electrons are emitted by the cathode C and collected by the plate A , called anode or collector. The two electrodes are connected to a high tension battery, a potential divider arrangement and a microammeter μA . The electrode C is connected to the midpoint of the potential divider arrangement, while electrode A can be given a positive or negative potential (as desired) with respect to plate C . Voltmeter V measures the potential difference applied between the two electrodes. When monochromatic radiations of suitable frequency fall on the plate C , electrons are emitted which are collected by the plate A . So a current, called *photoelectric current*, flows in the outer circuit which is measured by the microammeter μA .

1. **Effect of intensity of light on photoelectric current.** If we allow radiations of a fixed frequency to fall on plate P and the accelerating potential difference between the two electrodes is kept fixed, then the photoelectric current is found to increase linearly with the intensity of incident radiation, as shown in Fig. 11.4. Since the photoelectric current is directly proportional to the number of photoelectrons emitted per second, this implies that *the number of photoelectrons emitted per second is proportional to the intensity of incident radiation*.

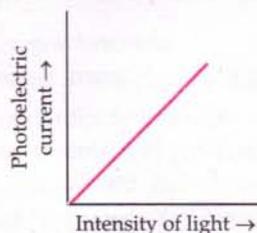


Fig. 11.4 Photoelectric current vs. intensity of incident radiation.

2. Effect of potential. As shown in Fig. 11.5, if we keep the intensity I_1 and the frequency of incident radiation fixed, and increase the positive potential (called *accelerating potential*) on plate A gradually, it is found that the photoelectric current increases with the increase in accelerating potential till a stage is reached when the photoelectric current becomes maximum and does not increase further with the increase in the accelerating potential. This maximum value of the photoelectric current is called the *saturation current*. At this stage, all the electrons emitted by the plate C are collected by the plate A.

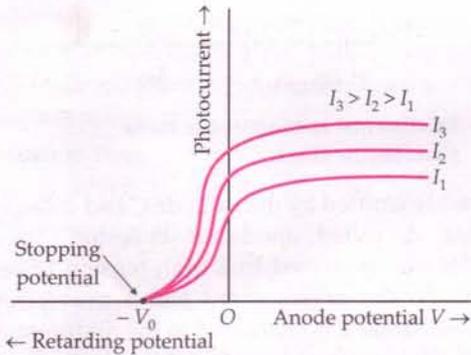


Fig. 11.5 Variation of photoelectric current with anode potential.

Now, if we apply a negative potential on plate A with respect to plate C and increase its magnitude gradually, it is seen that the photoelectric current decreases rapidly until it becomes zero for a certain value of negative potential on plate A. The value of the retarding potential at which the photoelectric current becomes zero is called *cut off or stopping potential* for the given frequency of the incident radiation.

For a given frequency of incident radiation, photoelectrons are emitted with all velocities ranging from zero to a certain maximum value v_{\max} . At the stopping potential V_0 , when no photoelectrons are emitted, the work done by stopping potential on the fastest electron must be equal to its kinetic energy. Hence

$$K_{\max} = \frac{1}{2} m v_{\max}^2 = e V_0$$

where m , e and v_{\max} are the mass, charge and maximum velocity of the electrons, respectively.

If we repeat the experiment with incident radiation of the same frequency but of higher intensity I_2 and I_3 ($I_3 > I_2 > I_1$), we find that the values of saturation currents have increased in proportion to the intensity of incident radiation, while the stopping potential is still the same. Thus, for a given frequency of incident

radiation, the stopping potential is independent of its intensity. This, in turn, implies that the maximum kinetic energy of the photoelectrons is independent of the intensity of incident radiation.

3. Effect of frequency of incident radiation on stopping potential. To study the effect of frequency on photoelectric effect, the intensity of incident radiation at each frequency is adjusted in such a way that the saturation current is same each time when the plate A is at a positive potential. The potential on the plate A is gradually reduced to zero and then increased in the negative direction till stopping potential is reached. The experiment is repeated with radiations of different frequencies.

As shown in Fig. 11.6, the value of stopping potential increases with the frequency of incident radiation. For frequencies $\nu_3 > \nu_2 > \nu_1$, the corresponding stopping potentials vary in the order $V_{03} > V_{02} > V_{01}$. This implies that greater the frequency of the incident radiation, greater is the maximum kinetic energy of the photoelectrons and hence greater is the retarding potential required to stop such electrons completely.

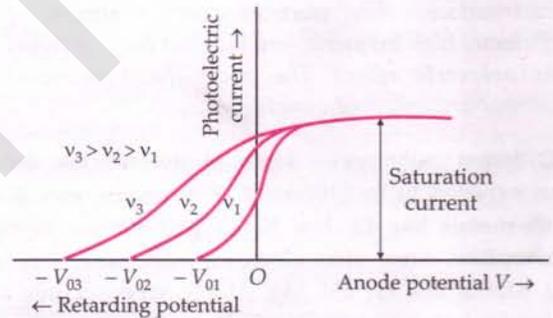


Fig. 11.6 Variation of photoelectric current with anode potential for different frequencies of incident radiation.

If we plot a graph between the frequency of incident radiation and the corresponding stopping potential for different metals, we get straight line

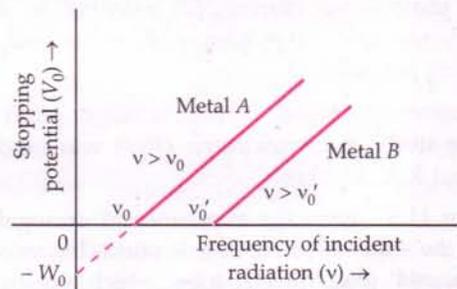


Fig. 11.7 Variation of stopping potential with frequency of incident radiation.

graphs, as shown in Fig. 11.7. These graphs reveal the following facts :

- (i) The stopping potential increases linearly with the frequency ν of the incident radiation for a given photosensitive material.
- (ii) There exists a certain minimum cut-off frequency for which the stopping potential is zero.

The minimum value of the frequency of incident radiation below which the photoelectric emission stops altogether is called **threshold frequency**.

- (iii) For two different metals A and B, these graphs are parallel straight lines i.e., they have same slope. But the threshold frequencies are different for the two metals.

The above observations imply two important facts :

- (i) The maximum kinetic energy of the photoelectrons increases linearly with the frequency of the incident radiation, but is independent of its intensity.
- (ii) For a frequency ν of the incident radiation less than the threshold frequency ν_0 , no photoelectric emission is possible, howsoever large is the intensity of incident radiation.

Once the frequency of incident light exceeds the threshold frequency, the photoelectric emission starts *instantaneously*, even if the light is very dim. The time interval between the incidence of radiation and the emission of electrons is of the order of 10^{-9} s or ns (nanosecond).

11.5 LAWS OF PHOTOELECTRIC EMISSION

5. State the laws of photoelectric emission.

Laws of photoelectric emission. On the basis of the experimental results on photoelectric effect, Lenard and Millikan gave the following laws of photoelectric emission :

1. For a given photosensitive material and frequency of incident radiation, (above the threshold frequency), the photoelectric current is directly proportional to the intensity of light. The saturation current is directly proportional to the intensity of incident radiation.
2. For a given photosensitive material, there exists a certain minimum cut-off frequency below which no photoelectrons are emitted, howsoever high is the intensity of incident radiation. This frequency is called **threshold frequency**.
3. Above the threshold frequency, the stopping potential or equivalently the maximum kinetic energy of the

photoelectrons is directly proportional to the frequency of incident radiation, but is independent of its intensity.

4. The photoelectric emission is an instantaneous process. The time lag between the incidence of light radiation and the emission of photoelectrons is very small, even less than 10^{-9} s.

11.6 FAILURE OF CLASSICAL WAVE THEORY

6. Explain why photoelectric effect cannot be explained on the basis of wave nature of light.

Failure of wave theory to explain photoelectric effect. According to wave theory, light is an electromagnetic wave consisting of electric and magnetic fields with continuous distribution of energy over the region over which the wave extends. This wave picture of light could not explain the basic features of light as explained below.

1. According to the wave theory, when a wavefront of light strikes a metal surface, the free electrons at the surface absorb the radiant energy continuously. Greater the intensity of incident radiation, greater are the amplitudes of electric and magnetic fields, and greater is the energy density of the wave. Hence higher intensity should liberate photoelectrons with greater kinetic energy. But this is contrary to the experimental result that the maximum kinetic energy of the photoelectrons does not depend on the intensity of incident radiation.

2. No matter what the frequency of incident radiation is, a light wave of sufficient intensity (over a sufficient time) should be able to impart enough energy required to eject the electrons from the metal surface. Thus the wave theory fails to explain the existence of threshold frequency.

3. The energy of light wave is smoothly and evenly distributed across its advancing wavefront. Each electron intercepts an insignificantly small amount of this energy and so it should require a *finite time* to escape from metal surface. But actually, the emission is almost instantaneous.

11.7 EINSTEIN'S THEORY OF PHOTOELECTRIC EFFECT

7. Explain Einstein's theory of photoelectric effect.

Einstein's theory of photoelectric effect. In 1905, Einstein explained photoelectric effect on the basis of Planck's quantum theory according to which a light radiation travels in the form of discrete photons. The energy of each photon is $h\nu$, where h is Planck's constant and ν is the frequency of light.

The main points of the Einstein's theory of photoelectric effect are :

1. Photoelectric emission is the result of interaction of two particles – one a photon of incident radiation and the other an electron of photosensitive metal.

2. The free electrons are bound within the metal due to restraining forces on the surface. The minimum energy required to liberate an electron from the metal surface is called work function W_0 of the metal.

3. Each photon interacts with one electron. The energy $h\nu$ of the incident photon is used up in *two* parts :

- a part of the energy of the photon is used in liberating the electron from the metal surface, which is equal to the work function W_0 of the metal, and
- the remaining energy of the photon is used in imparting kinetic energy to the ejected electron.

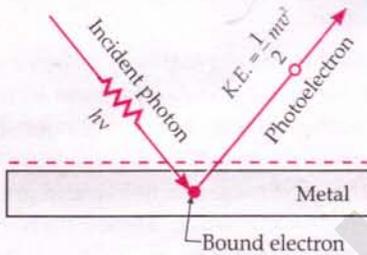


Fig. 11.8 Photoelectric emission.

4. Very few (<1%) photons, whose energies are greater than W_0 , are capable of ejecting the photo- electrons.

By the conservation of energy,
Energy of the incident photon

$$= \text{Maximum K.E. of photoelectron} \\ + \text{Work function}$$

or
$$h\nu = \frac{1}{2}mv_{\max}^2 + W_0$$

or
$$K_{\max} = \frac{1}{2}mv_{\max}^2 = h\nu - W_0 \quad \dots(1)$$

If the incident photon is of threshold frequency ν_0 , then its energy $h\nu_0$ is just sufficient to free the electron from the metal surface and does not give it any kinetic energy. So $h\nu_0 = W_0$. Hence

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = h\nu - h\nu_0 = h(\nu - \nu_0) \quad \dots(2)$$

Equations (1) and (2) are called *Einstein's photoelectric equations* and can be used to explain the laws of photoelectric effect as follows :

1. **Explanation of effect of intensity.** The increase of intensity means the increase in the number of photons

striking the metal surface per unit time. As each photon ejects only one electron, so the number of ejected photoelectrons increases with the increase in intensity of incident radiation.

2. **Explanation of threshold frequency.** If $\nu < \nu_0$ i.e., the frequency of incident radiation is less than the threshold frequency, the kinetic energy of photoelectrons becomes negative. This has no physical meaning. So photoelectric emission does not occur below the threshold frequency.

3. **Explanation of kinetic energy.** If $\nu > \nu_0$, then

$$K_{\max} = \frac{1}{2}mv_{\max}^2 \propto \nu$$

i.e., above the threshold frequency, the maximum kinetic energy of the electrons increases linearly with the frequency ν of the incident radiation. Moreover, the increase in intensity increases only the number of incident photons and not their energy. Hence the maximum kinetic energy of the photoelectrons is independent of the intensity of incident radiation.

4. **Explanation of time lag.** Photoelectric emission is the result of an elastic collision between a photon and an electron. Thus the absorption of energy from a photon by a free electron inside the metal is a single event which involves transfer of energy in one lump instead of the continuous absorption of energy as in the wave theory of light. Hence there is no time lag between the incidence of a photon and the emission of a photoelectron.

11.8 PARTICLE NATURE OF LIGHT : THE PHOTON

8. Write the basic features of the photon picture of electromagnetic radiation on which Einstein's Theory of photoelectric effect is based.

The photon picture of electromagnetic radiation. The observations on photoelectric effect clearly indicate that when light interacts with matter, it behaves as if it was made of discrete packets of energy, called quanta. Each quantum of light has energy $h\nu$ and momentum ($h\nu/c$). The fact that a light quantum has definite energy as well as momentum allows us to associate a particle with it. This particle was called *photon*. The particle nature of electromagnetic radiation was further confirmed in 1924, by the experiment of A.H. Compton on scattering of X-rays by electrons. The phenomenon of increase in the wavelength of X-ray photons scattered by the striking electrons is called *Compton effect*. In 1921, Einstein was awarded the Noble Prize in Physics for his contribution to theoretical physics and the photoelectric effect.

Some basic features of the photon picture of electromagnetic radiation are as follows :

1. In the interaction of radiation with matter, radiation behaves as if it is made of particles, called photons.
2. Each photon has energy $E(=h\nu)$ and momentum $p(=h\nu/c=h/\lambda)$, and speed c (= speed of light) in vacuum.
3. All photons of light of a particular frequency ν , or wavelength λ , have the same energy ($E = h\nu = hc/\lambda$) and momentum $p(=h\nu/c = h/\lambda)$, independent of the intensity of the radiation.
4. Photons are electrically neutral and are not deflected by electric and magnetic fields.
5. In a photon-particle (or photon-electron) collision, the total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.
6. If the intensity of light of a given wavelength is increased, there is an increase in the number of photons incident on a given area in a given time. But the energy of each photon remains the same.

NOTE Some other properties of photons are as follows :

1. The frequency of a photon does not change as it travels from one medium to another.
2. The speed of a photon changes as it travels through different media due to the change in its wavelength.
3. The rest mass of a photon is zero *i.e.*, a photon cannot exist at rest. For a photon, $v = c$, so

$$m_0 = m\sqrt{1 - \frac{v^2}{c^2}} = m\sqrt{1 - \frac{c^2}{c^2}} = 0$$

4. From Einstein's mass-energy relationship, the equivalent mass m of a photon is given by

$$E = mc^2 = h\nu \quad \text{or} \quad m = \frac{h\nu}{c^2}$$

5. The linear momentum of a photon is

$$p = mc = \frac{h\nu}{c} = \frac{h}{\lambda}$$

11.9 DETERMINATION OF PLANCK'S CONSTANT AND WORK FUNCTION

9. Draw a graph showing the variation of stopping potential with the frequency of incident radiation in relation to photoelectric effect. Deduce an expression for the slope of this graph using Einstein's photoelectric equation.

Determination of Planck's constant and work function. According to Einstein's photoelectron equation, the maximum K.E. of a photoelectron is given by

$$K_{\max} = h\nu - W_0$$

If V_0 is the stopping potential, then

$$K_{\max} = eV_0$$

$$\therefore eV_0 = h\nu - W_0$$

or

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{W_0}{e} \quad \dots(1)$$

We compare this equation with the straight line equation,

$$y = mx + c$$

It follows from equation (1) that V_0 versus ν graph is a straight line, as shown in Fig. 11.9.

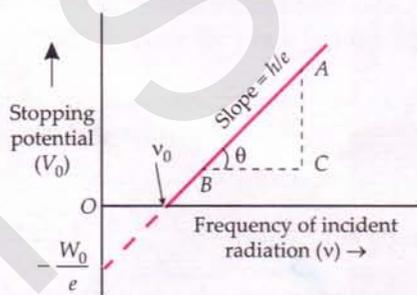


Fig. 11.9 V_0 versus ν graph for a photo-sensitive material.

Clearly, slope of V_0 - ν graph $= \frac{h}{e}$

To determine the slope, take two points A and B on the straight line graph. Then

$$m = \tan \theta = \frac{AC}{BC} = \frac{h}{e}$$

$$\therefore h = e \times \frac{AC}{BC} = e \times \text{slope of } V_0\text{-}\nu \text{ graph}$$

Thus, the Planck's constant h can be determined.

Moreover, the intercept on vertical axis $= -\frac{W_0}{e}$

$$\therefore W_0 = e \times \text{Magnitude of the intercept on vertical axis.}$$

In this way, the work function W_0 can be determined.

By measuring the slope of V_0 - ν graph for sodium and using the known value of e , R.A. Millikan precisely determined the values of h and W_0 (for sodium). These values agreed well with the values known from other experiments. This led to the acceptance of Einstein's particle or photon picture of electromagnetic radiation. R.A. Millikan was awarded the Noble Prize in 1923 for his work on the determination of e and photoelectric effect.

Examples based on

Photons and Photoelectric Effect

Formulae Used

1. Energy of a photon, $E = hv = \frac{hc}{\lambda}$.
2. Number of photons emitted per second, $N = \frac{P}{E}$.
3. Momentum of photon, $p = mc = \frac{hv}{c} = \frac{h}{\lambda}$.
4. Equivalent mass of a photon, $m = \frac{hv}{c^2}$.
5. Work function, $W_0 = hv_0 = \frac{hc}{\lambda_0}$.
6. Kinetic energy of photoelectrons is given by Einstein's photoelectric equation,

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = hv - W_0 = h(v - v_0) = h\left[\frac{c}{\lambda} - \frac{c}{\lambda_0}\right]$$
7. If V_0 is the stopping potential, the maximum kinetic energy of the ejected photo electrons,

$$K = \frac{1}{2}mv_{\max}^2 = eV_0$$
8. Intensity of radiation = $\frac{\text{Energy}}{\text{Area} \times \text{time}} = \frac{\text{Power}}{\text{Area}}$

Incident power = Incident intensity \times area.

Units Used

Energy E is in joule, power P in watt, momentum p in kg ms^{-1} , work function W_0 is in joule or eV, stopping potential V_0 in volt, wavelengths λ and λ_0 in metre, frequencies ν and ν_0 in Hz.

Constants Used

Planck's constant, $h = 6.63 \times 10^{-34} \text{ Js}$
 Speed of light in vacuum, $c = 3 \times 10^8 \text{ ms}^{-1}$.

Conversion Used

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

Example 1. What is the frequency of a photon whose energy is 66.3 eV? [Punjab 97]

Solution. Here

$$E = 66.3 \text{ eV} = 66.3 \times 1.6 \times 10^{-19} \text{ J}$$

Frequency,

$$\nu = \frac{E}{h} = \frac{66.3 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.6 \times 10^{16} \text{ Hz.}$$

Example 2. If a light of wavelength 4950 Å is viewed as a continuous flow of photons, what is the energy of each photon in eV? Given $h = 6.6 \times 10^{-34} \text{ Js}$, $c = 3 \times 10^8 \text{ ms}^{-1}$. [ISCE 98]

Solution. Here $\lambda = 4950 \text{ Å} = 4950 \times 10^{-10} \text{ m}$

Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4950 \times 10^{-10}} = 4.0 \times 10^{-19} \text{ J}$$

$$= \frac{4.0 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 2.5 \text{ eV.}$$

Example 3. Monochromatic light of frequency $6.0 \times 10^{14} \text{ Hz}$ is produced by a laser. The power emitted is $2.0 \times 10^{-3} \text{ W}$. (i) What is the energy of each photon in the light? (ii) How many photons per second, on the average, are emitted by the source? [NCERT ; CBSE D 14]

Solution. Here $\nu = 6.0 \times 10^{14} \text{ Hz}$, $P = 2.0 \times 10^{-3} \text{ W}$

(i) Energy of each photon,

$$E = hv = 6.63 \times 10^{-34} \times 6.0 \times 10^{14}$$

$$= 3.98 \times 10^{-19} \text{ J.}$$

(ii) If N is the number of photons emitted per second by the source, then

Power transmitted in the beam

$$= N \times \text{energy of each photon}$$

or $P = NE$

$$\therefore N = \frac{P}{E} = \frac{2.0 \times 10^{-3} \text{ W}}{3.98 \times 10^{-19} \text{ J}}$$

or $N = 5.0 \times 10^{15} \text{ photons per second.}$

Example 4. A monochromatic source, emitting light of wavelength, 600 nm, has a power output of 66 W. Calculate the number of photons emitted by this source in 2 minutes. [CBSE Sample Paper 13]

Solution. Energy of each photon,

$$E = hv = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6 \times 10^{-7}} = 3.3 \times 10^{-19} \text{ J}$$

Power of the source, $P = 66 \text{ W}$

Number of photons emitted per second,

$$N = \frac{P}{E} = \frac{66}{3.3 \times 10^{-19}} = 2 \times 10^{20}$$

Total number of photons emitted by the source in 2 minutes,

$$n = N \times 2 \times 60$$

$$= 2 \times 10^{20} \times 120 = 2.4 \times 10^{22} \text{ photons.}$$

Example 5. If 5% of the energy supplied to an incandescent light bulb is radiated as visible light, how many visible light photons are emitted by 100 watt bulb? Assume wavelength of all visible photons to be 5600 Å. Given $h = 6.625 \times 10^{-34} \text{ Js}$. [VMMC PMT 15]

Solution. Here $\lambda = 5600 \text{ Å} = 5600 \times 10^{-10} \text{ m}$

Energy of one photon

$$= \frac{hc}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{5600 \times 10^{-10}} = 3.5 \times 10^{-19} \text{ J}$$

A 100 W bulb supplies 100 J of energy per second.

∴ Energy released per second as visible photons

$$= \frac{100 \times 5}{100} = 5 \text{ J}$$

∴ Number of photons emitted per second as visible light

$$= \frac{5}{3.5 \times 10^{-19}} = 1.43 \times 10^{19}$$

Example 6. Energy from the sun is received on the earth at the rate of $2 \text{ cal cm}^{-2} \text{ min}^{-1}$. If average wavelength of solar light be taken as 5500 \AA , then how many photons are received on earth per cm^2 per minute? Take $1 \text{ cal} = 4.2 \text{ J}$, $c = 3 \times 10^8 \text{ ms}^{-1}$.

Solution. Rate of energy received from the sun

$$= 2 \text{ cal cm}^{-2} \text{ min}^{-1} = 2 \times 4.2 = 8.4 \text{ J cm}^{-2} \text{ min}^{-1}$$

Energy of a photon received from the sun is

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5500 \times 10^{-10}}$$

$$= 3.6 \times 10^{-19} \text{ J}$$

If n is the number of photons reaching the earth per cm^2 per minute, then their total energy will be $3.6 \times 10^{-19} n$ joule.

$$\therefore 3.6 \times 10^{-19} n = 8.4$$

$$\text{or } n = \frac{8.4}{3.6 \times 10^{-19}} = 2.3 \times 10^{19}$$

Example 7. For a photosensitive surface, work function is $3.3 \times 10^{-19} \text{ J}$. Taking Planck's constant to be $6.6 \times 10^{-34} \text{ Js}$, find threshold frequency. [ISCE 97]

Solution. Here $W_0 = 3.3 \times 10^{-19} \text{ J}$,

$$h = 6.6 \times 10^{-34} \text{ Js}$$

Threshold frequency,

$$\nu_0 = \frac{W_0}{h} = \frac{3.3 \times 10^{-19}}{6.6 \times 10^{-34}} = 5 \times 10^{14} \text{ Hz.}$$

Example 8. The work function of caesium is 2.14 eV . Find (a) the threshold frequency for caesium, and (b) the wavelength of the incident light if the photocurrent is brought to zero by a stopping potential of 0.60 eV .

[NCERT ; CBSE OD 04 ; F 04]

Solution. (a) For the minimum, cut-off or threshold frequency,

Energy $h\nu_0$ of incident photon = Work function W_0

$$\begin{aligned} \therefore \nu_0 &= \frac{W_0}{h} \\ &= \frac{2.14 \text{ eV}}{6.63 \times 10^{-34} \text{ Js}} = \frac{2.14 \times 1.6 \times 10^{-19} \text{ J}}{6.63 \times 10^{-34} \text{ Js}} \\ &= 5.16 \times 10^{14} \text{ Hz.} \end{aligned}$$

(b) When photoelectric current becomes zero,

Maximum K.E. of photoelectron
= P.E. due to the stopping potential V_0

$$\text{or } K_{\text{max}} = eV_0$$

$$\text{or } \frac{hc}{\lambda} - W_0 = eV_0$$

$$\text{or } \lambda = \frac{hc}{eV_0 + W_0}$$

$$= \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{0.60 \text{ eV} + 2.14 \text{ eV}}$$

$$= \frac{19.89 \times 10^{-26} \text{ Jm}}{2.74 \text{ eV}} = \frac{19.89 \times 10^{-26} \text{ Jm}}{274 \times 1.6 \times 10^{-19} \text{ J}}$$

$$= 453.7 \times 10^{-9} \text{ m} \approx 453.7 \text{ nm.}$$

Example 9. The following table gives the values of work function for a few photosensitive metals :

S.No.	Metal	Work Function (eV)
1.	Na	1.92
2.	K	2.15
3.	Mo	4.17

If each of these metals is exposed to radiations of wavelength 300 nm , which of them will not emit photoelectrons and why? [CBSE Sample Paper 08]

Solution. Energy of an incident photon,

$$\begin{aligned} E &= \frac{hc}{\lambda} \\ &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}} \text{ J} = 6.6 \times 10^{-19} \text{ J} \\ &= \frac{6.6 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.16 \text{ eV} \end{aligned}$$

Mo will not emit photoelectrons because the energy of incident photon is less than the work function of Mo.

Example 10. Photoelectrons are emitted from a metal surface when UV light of wavelength $\lambda = 300 \text{ nm}$ is incident on it. The minimum negative potential required to stop the emission of electrons is 0.54 V . Calculate :

- the energy of the incident photons
- the maximum kinetic energy of the photoelectrons emitted
- the work function of the metal.

Express all answers in eV.

[ISCE 03]

Solution. Here $\lambda = 300 \text{ nm} = 3 \times 10^{-7} \text{ m}$

$$V_0 = 0.54 \text{ V}$$

(i) Energy of incident photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3 \times 10^{-7}} = 6.63 \times 10^{-19} \text{ J}$$

$$= \frac{6.63 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 4.14 \text{ eV.}$$

(ii) $K_{\max} = eV_0 = 0.54 \text{ eV.}$ (iii) Again, $K_{\max} = hv - W_0$

$$\therefore W_0 = hv - K_{\max}$$

$$= 4.14 - 0.54 = 3.6 \text{ eV.}$$

Example 11. Light of wavelength 5000 \AA falls on a metal surface of work function 1.9 eV . Find (i) the energy of photons in eV (ii) the K.E. of photoelectrons and (iii) the stopping potential. [Punjab 02]

Solution. Here

$$\lambda = 5000 \text{ \AA} = 5 \times 10^{-7} \text{ m}, \quad W_0 = 1.9 \text{ eV}$$

(i) Energy of a photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5 \times 10^{-7}} \text{ J}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5 \times 10^{-7} \times 1.6 \times 10^{-19}} \text{ eV} = 2.4825 \text{ eV.}$$

(ii) K.E. of a photoelectron

$$= hv - W_0 = 2.4825 - 1.9 = 0.5825 \text{ eV.}$$

(iii) Let V_0 be the stopping potential. Then

$$eV_0 = \frac{1}{2} mv^2 = \text{K.E. of a photoelectron}$$

or

$$V_0 = \frac{\text{K.E. of a photoelectron}}{e}$$

$$= \frac{0.5825 \text{ eV}}{e} = 0.5825 \text{ V.}$$

Example 12. The work function for caesium is 1.8 eV . Light of 5000 \AA is incident on it. Calculate (i) threshold frequency and threshold wavelength, (ii) maximum kinetic energy of the emitted electrons, (iii) maximum velocity of the emitted electrons, (iv) if the intensity of the incident light be doubled, then what will be the maximum kinetic energy of the emitted electrons? Given : $h = 6.6 \times 10^{-34} \text{ Js}$, $m_e = 9 \times 10^{-31} \text{ kg}$, $c = 3 \times 10^8 \text{ ms}^{-1}$.

Solution. Here $W_0 = 1.8 \text{ eV} = 1.8 \times 1.6 \times 10^{-19} \text{ J}$

$$\lambda = 5000 \text{ \AA} = 5000 \times 10^{-10} \text{ m}$$

(i) Threshold frequency,

$$v_0 = \frac{W_0}{h} = \frac{1.8 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= 4.363 \times 10^{14} \text{ Hz}$$

Threshold wavelength,

$$\lambda_0 = \frac{c}{v_0} = \frac{3 \times 10^8}{4.363 \times 10^{14}}$$

$$= 6.875 \times 10^{-7} \text{ m.}$$

(ii) Maximum K.E. of emitted electrons,

$$K_{\max} = \frac{hc}{\lambda} - W_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} - 1.8 \times 1.6 \times 10^{-19}$$

$$= 3.96 \times 10^{-19} - 2.88 \times 10^{-19}$$

$$= 1.08 \times 10^{-19} \text{ J.}$$

(iii) \therefore Maximum velocity of emitted electrons,

$$v_{\max} = \sqrt{\frac{2 K_{\max}}{m}} = \sqrt{\frac{2 \times 1.08 \times 10^{-19}}{9 \times 10^{-31}}}$$

$$\left[\because K_{\max} = \frac{1}{2} mv_{\max}^2 \right]$$

$$= 4.9 \times 10^5 \text{ ms}^{-1}.$$

(iv) The kinetic energy of the emitted electrons does not depend upon the intensity of the incident light. Hence if the intensity of the incident light is doubled, the maximum kinetic energy of the emitted electrons remains unchanged.

Example 13. Find the frequency of light, which ejects electrons from a metal surface fully stopped by a retarding potential of 3 V . The photoelectric effect begins in this metal at frequency of $6 \times 10^{14} \text{ s}^{-1}$. Find the work function of this metal. [Himachal 96]

Solution. Here $V_0 = 3 \text{ V}$, $v_0 = 6 \times 10^{14} \text{ s}^{-1}$

Using Einstein's photoelectric equation,

$$hv = hv_0 + \frac{1}{2} mv_{\max}^2$$

$$\text{But } \frac{1}{2} mv_{\max}^2 = eV_0$$

$$\therefore hv = hv_0 + eV_0$$

or

$$v = v_0 + \frac{eV_0}{h}$$

$$= 6 \times 10^{14} + \frac{1.6 \times 10^{-19} \times 3}{6.6 \times 10^{-34}}$$

$$= 6 \times 10^{14} + 7.25 \times 10^{14}$$

$$= 13.25 \times 10^{14} \text{ s}^{-1}.$$

Work function,

$$W_0 = hv_0 = 6.6 \times 10^{-34} \times 6 \times 10^{14} \text{ J}$$

$$= \frac{6.6 \times 10^{-34} \times 6 \times 10^{14}}{1.6 \times 10^{-19}} = 2.48 \text{ eV.}$$

Example 14. For photoelectric effect in sodium, Fig. 11.10 shows the plot of cut-off voltage versus frequency of incident radiation. Calculate :

- the threshold frequency.
- the work function for sodium. [CBSE D 95]

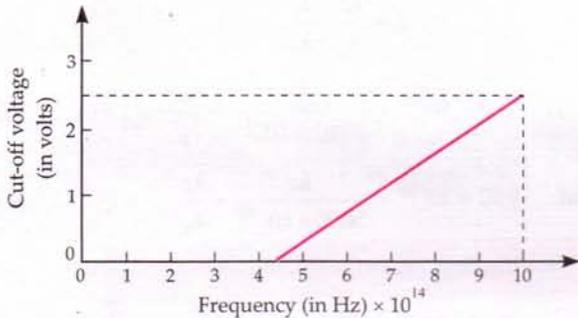


Fig. 11.10

Solution. (i) From the given graph, threshold frequency is

$$\nu_0 = 4.5 \times 10^{14} \text{ Hz.}$$

- Work function of the metal is

$$\begin{aligned} W_0 &= h\nu_0 = 6.6 \times 10^{-34} \times 4.5 \times 10^{14} \\ &= 2.97 \times 10^{-19} \text{ J} = 1.86 \text{ eV.} \end{aligned}$$

Example 15. Using the graph shown in Fig. 11.11 for stopping potential vs. the incident frequency of photons, calculate Planck's constant. [CBSE D 15C]

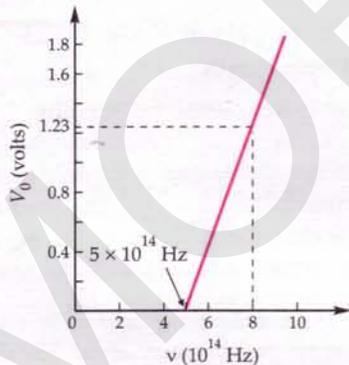


Fig. 11.11

Solution. Einstein's photoelectric equation is

$$eV_0 = h\nu - W_0$$

On differentiation, we get

$$e\Delta V_0 = h\Delta\nu$$

$$\begin{aligned} \therefore h &= \frac{\Delta V_0}{\Delta\nu} \cdot e = \frac{1.23 - 0}{(8 - 5) \times 10^{14}} \times 1.6 \times 10^{-19} \text{ Js} \\ &= 6.56 \times 10^{-34} \text{ Js.} \end{aligned}$$

Example 16. If photoelectrons are to be emitted from a potassium surface with a speed of $6 \times 10^6 \text{ ms}^{-1}$, what frequency of radiation must be used? (Threshold frequency for potassium is $4.22 \times 10^{14} \text{ Hz}$, $h = 6.6 \times 10^{-34} \text{ Js}$ and $m_e = 9.1 \times 10^{-31} \text{ kg}$). [CBSE Sample Paper 98]

Solution. Here $v = 6 \times 10^6 \text{ ms}^{-1}$,
 $\nu_0 = 4.22 \times 10^{14} \text{ Hz}$, $h = 6.6 \times 10^{-34} \text{ Js}$,
 $m_e = 9.1 \times 10^{-31} \text{ kg}$

From Einstein's photoelectric equation,

$$\text{K.E.} = \frac{1}{2} m v^2 = h(\nu - \nu_0)$$

$$\begin{aligned} \therefore \nu &= \frac{1}{2} \cdot \frac{m v^2}{h} + \nu_0 \\ &= \frac{1}{2} \times \frac{9.1 \times 10^{-31} \times (6 \times 10^6)^2}{6.6 \times 10^{-34}} + 4.22 \times 10^{14} \\ &= 2.48 \times 10^{14} + 4.22 \times 10^{14} = 6.7 \times 10^{14} \text{ Hz.} \end{aligned}$$

Example 17. A sheet of silver is illuminated by monochromatic ultraviolet light of wavelength = 1810 \AA . What is the maximum energy of the emitted electron? Threshold wavelength of silver is 2640 \AA .

Solution. Here $\lambda = 1810 \text{ \AA} = 1810 \times 10^{-10} \text{ m}$
 $\lambda_0 = 2640 \text{ \AA} = 2640 \times 10^{-10} \text{ m}$

From Einstein's photoelectric equation, the maximum energy of the emitted photoelectron is

$$\begin{aligned} K_{\max} &= h\nu - W_0 = h\nu - h\nu_0 \\ &= h \left[\frac{c}{\lambda} - \frac{c}{\lambda_0} \right] = hc \left[\frac{1}{\lambda} - \frac{1}{\lambda_0} \right] \\ &= 6.63 \times 10^{-34} \times 3 \times 10^8 \\ &\quad \times \left[\frac{1}{1810 \times 10^{-10}} - \frac{1}{2640 \times 10^{-10}} \right] \text{ J} \\ &= \frac{6.63 \times 3 \times 10^{-16}}{1.6 \times 10^{-19}} \left[\frac{2640 - 1810}{1810 \times 2640} \right] \text{ eV} \\ &= \frac{6.63 \times 3 \times 830 \times 10^3}{1.6 \times 1810 \times 2640} \text{ eV} = 2.16 \text{ eV.} \end{aligned}$$

Example 18. By how much would the stopping potential for a given photosensitive surface go up if the frequency of the incident radiations were to be increased from $4 \times 10^{15} \text{ Hz}$ to $8 \times 10^{15} \text{ Hz}$? Given $h = 6.4 \times 10^{-34} \text{ Js}$, $e = 1.6 \times 10^{-19}$ and $c = 3 \times 10^8 \text{ ms}^{-1}$. [CBSE Sample Paper 08]

Solution. According to Einstein's photoelectric equation,

$$eV_0 = h\nu - W_0$$

$$\begin{aligned} \therefore eV_{01} &= hv_1 - W_0 \quad \text{and} \quad eV_{02} = hv_2 - W_0 \\ \text{or } e(V_{02} - V_{01}) &= h(v_2 - v_1) \\ \text{or } V_{02} - V_{01} &= \frac{h(v_2 - v_1)}{e} \\ &= \frac{6.4 \times 10^{-34} \times (8 - 4) \times 10^{15}}{1.6 \times 10^{-19}} = 16 \text{ V.} \end{aligned}$$

Example 19. Determine Planck's constant h if photoelectrons emitted from a certain metal surface by light of frequency 2.2×10^{15} Hz are fully retarded by a reverse potential of 6.6 V and those emitted by light of frequency 4.6×10^{15} Hz are stopped by a reverse potential of 16.5 V.

Solution. From Einstein's photoelectric equation, if V_0 is the stopping potential, then maximum energy of the emitted photoelectron is

$$K_{\max} = eV_0 = h(\nu - \nu_0)$$

By substituting the given values, we get

$$6.6 e = h [2.2 \times 10^{15} - \nu_0]$$

$$\text{and } 16.5 e = h [4.6 \times 10^{15} - \nu_0]$$

$$\therefore 16.5 e - 6.6 e = h [4.6 \times 10^{15} - 2.2 \times 10^{15}]$$

$$\text{or } 9.9 \times 1.6 \times 10^{-19} = h \times 2.4 \times 10^{15}$$

$$\text{Hence } h = \frac{9.9 \times 1.6 \times 10^{-19}}{2.4 \times 10^{15}} = 6.6 \times 10^{-34} \text{ Js.}$$

Example 20. The photoelectric threshold wavelength for a metal is 10,000 Å. When light of wavelength 5461 Å is incident on it, the retarding potential in Millikan's experiment is 1.02 V. Calculate the value of Planck's constant.

Solution. If V_0 is the stopping potential, then Einstein's photoelectric equation can be written as

$$\frac{1}{2} m v_{\max}^2 = eV_0 = h(\nu - \nu_0)$$

$$\text{or } eV_0 = h \left[\frac{c}{\lambda} - \frac{c}{\lambda_0} \right] = hc \left[\frac{\lambda_0 - \lambda}{\lambda_0 \lambda} \right]$$

$$\begin{aligned} h &= \frac{e \lambda_0 \lambda V_0}{(\lambda_0 - \lambda) c} \\ &= \frac{1.6 \times 10^{-19} \times 10000 \times 10^{-10} \times 5461 \times 10^{-10} \times 1.02}{(10000 - 5461) \times 10^{-10} \times 3 \times 10^8} \text{ Js} \\ &= \frac{1.6 \times 5461 \times 1.02 \times 10^{-37}}{4539 \times 3} \text{ Js} \\ &= 6.554 \times 10^{-34} \text{ Js.} \end{aligned}$$

Example 21. A photon of wavelength 3310 Å falls on a photo-cathode and an electron of energy 3×10^{-19} J is ejected. If the wavelength of the incident photon is changed to 5000 Å, the energy of the ejected electron is 9.72×10^{-20} J.

Calculate the value of Planck's constant and threshold wavelength of the photon. [CBSE Sample Paper 98]

Solution. When a photon of wavelength λ is incident on a photocathode, the energy of the ejected electron is given by

$$E = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$\therefore 3 \times 10^{-19} = \frac{hc}{3310 \times 10^{-10}} - \frac{hc}{\lambda_0} \quad \dots(i)$$

$$\text{and } 9.72 \times 10^{-20} = \frac{hc}{5000 \times 10^{-10}} - \frac{hc}{\lambda_0} \quad \dots(ii)$$

Subtracting (ii) from (i), we get

$$(3 - 0.972) \times 10^{-19} = \frac{hc}{10^{-10}} \left(\frac{1}{3310} - \frac{1}{5000} \right)$$

$$\text{or } 2.028 \times 10^{-19} = \frac{h \times 3 \times 10^8}{10^{-10}} \times \frac{1690}{3310 \times 5000}$$

$$\begin{aligned} \therefore h &= \frac{2.028 \times 10^{-19} \times 10^{-10} \times 3310 \times 5000}{3 \times 10^8 \times 1690} \\ &= 6.62 \times 10^{-34} \text{ Js.} \end{aligned}$$

$$\text{Now } W_0 = \frac{hc}{\lambda} - E$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3310 \times 10^{-10}} - 3 \times 10^{-19}$$

$$= (6 - 3) \times 10^{-19} = 3 \times 10^{-19} \text{ J}$$

Threshold wavelength,

$$\begin{aligned} \lambda_0 &= \frac{hc}{W_0} \\ &= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3 \times 10^{-19}} \\ &= 6.62 \times 10^{-7} \text{ m} = 6620 \text{ Å.} \end{aligned}$$

Example 22. The wavelength of light in the visible region is about 390 nm for violet colour, about 550 nm (average wavelength) for yellow-green colour and about 760 nm for red colour.

- What are the energies of photons in (eV) at the (i) violet end, (ii) average wavelength, yellow-green colour, (iii) red end of the visible spectrum? [Take $h = 6.63 \times 10^{-34}$ Js and $1 \text{ eV} = 1.6 \times 10^{-19}$ J]
- From which of the photosensitive materials with work functions listed in Table 11.1 and using the results of (i), (ii) and (iii) of (a), can you build a photoelectric device that operates with visible light?

Solution. (a) Energy of the incident photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{\lambda} \\ = \frac{1.989 \times 10^{-25} \text{ Jm}}{\lambda}$$

(i) For violet light, $\lambda_1 = 390 \text{ nm}$ (lower wavelength end), the energy of incident photon is

$$E_1 = \frac{1.989 \times 10^{-25} \text{ Jm}}{390 \times 10^{-9} \text{ m}} = 5.10 \times 10^{-19} \text{ J} \\ = \frac{5.10 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 3.19 \text{ eV.}$$

(ii) For yellow-green light, $\lambda_2 = 550 \text{ nm}$ (average wavelength), the energy of incident photon is

$$E_2 = \frac{1.989 \times 10^{-25} \text{ Jm}}{550 \times 10^{-9} \text{ m}} \\ = 3.62 \times 10^{-19} \text{ J} = 2.26 \text{ eV.}$$

(iii) For red light, $\lambda_3 = 760 \text{ nm}$ (higher wavelength end), the energy of incident photon is

$$E_3 = \frac{1.989 \times 10^{-25} \text{ Jm}}{760 \times 10^{-9} \text{ m}} \\ = 2.62 \times 10^{-19} \text{ J} = 1.64 \text{ V.}$$

(b) A photoelectric device will operate when

Energy E of the incident photon $>$ Work function W_0

Thus a photoelectric device will operate with violet light ($E = 3.19 \text{ eV}$) by using photosensitive materials like Na ($W_0 = 2.75 \text{ eV}$), K ($W_0 = 2.30 \text{ eV}$) and Cs ($W_0 = 2.14 \text{ eV}$)

The photoelectric device will operate with yellow green light ($E = 2.26 \text{ eV}$) by using Cs ($W_0 = 2.14 \text{ eV}$) only.

The photoelectric device will not operate with red light ($E = 1.64 \text{ eV}$) by using any of the photosensitive materials.

Problems For Practice

- Calculate the energy and momentum of a photon of wavelength 6600 \AA . (Ans. $3 \times 10^{-19} \text{ J}$, $10^{-27} \text{ kg ms}^{-1}$)
- Calculate the frequency associated with a photon of energy $3.3 \times 10^{-20} \text{ J}$. [Haryana 02] (Ans. $0.5 \times 10^{14} \text{ Hz}$)
- The wavelength of a spectral line is 4000 \AA . Calculate its frequency and energy. [CBSE D 94] (Ans. $7.5 \times 10^{14} \text{ Hz}$, $4.95 \times 10^{-19} \text{ J}$)
- An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.66 \AA . What is the maximum energy of the photon in the radiation? [CBSE OD 90] (Ans. $3.0 \times 10^{-15} \text{ J}$)
- Find the number of photons emitted per minute by a 25 W source of monochromatic light of wavelength 5000 \AA . [Punjab 04] (Ans. 3.78×10^{21})
- Find the photon energy in eV for an electromagnetic wave of wavelength 1 m . [Haryana 93] (Ans. $1.243 \times 10^{-6} \text{ eV}$)
- Calculate the frequency of a photon of energy 0.5 keV . Given $h = 6.62 \times 10^{-34} \text{ Js}$. [Haryana 03] (Ans. $1.2 \times 10^{17} \text{ Hz}$)
- Monochromatic light of frequency $5.0 \times 10^{14} \text{ Hz}$ is produced by a laser. The power emitted is $3.0 \times 10^{-3} \text{ W}$. Estimate the number of photons emitted per second on an average by the source. [CBSE D 14] (Ans. 9.1×10^{15} photons/s)
- A monochromatic source of light operating at 200 W emits 4×10^{20} photons per second. Find the wavelength of the light. (Ans. 400 nm)
- If a photoemissive surface has a threshold frequency of $4.6 \times 10^{14} \text{ Hz}$, calculate the energy of the photons in eV. Given $h = 6.6 \times 10^{-34} \text{ Js}$. [CBSE F 94 C] (Ans. 1.9 eV)
- Calculate the longest wavelength of the incident radiation, which will eject photoelectrons from a metal surface, whose work function is 3 eV . [Punjab 01] (Ans. 4137.5 \AA)
- A metal has a threshold wavelength of 6000 \AA . Calculate (i) threshold frequency (ii) work function of the metal in eV. Given $h = 6.62 \times 10^{-34} \text{ Js}$ and $e = 1.6 \times 10^{-19} \text{ C}$. [Ans. (i) $5 \times 10^{14} \text{ Hz}$ (ii) 2.07 eV]
- Calculate the threshold frequency of photon for photoelectric emission from a metal of work function 0.1 eV . [CBSE D 92] (Ans. $2.4 \times 10^{13} \text{ Hz}$)
- Work function of sodium is 2.3 eV . Does sodium show photoelectric emission for orange light ($\lambda = 6800 \text{ \AA}$)? Given $h = 6.63 \times 10^{-34} \text{ Js}$. [Haryana 94] (Ans. No)
- A metal sheet is given a negative charge of 11.2 nC . (i) How many photons of ultraviolet are required to completely discharge the metal sheet? (ii) What is the minimum amount of energy that must be absorbed by metal to affect this discharge?

Threshold frequency of metal = 4.5×10^{14} Hz and $h = 6.6 \times 10^{-34}$ Js. [Punjab 93]

[Ans. (i) 7×10^{10} , (ii) 1.856 eV]

16. Light of wavelength 5000 \AA falls on a metal surface of work function 2.01 eV. Find the kinetic energy of photoelectrons. [Punjab 01]

(Ans. 0.4725 eV)

17. Calculate the kinetic energy of a photoelectron (in eV) emitted on shining light of wavelength 6.2×10^{-6} m on a metal surface. The work function of the metal is 0.1 eV. [CBSE OD 92]

(Ans. 0.1 eV)

18. Photoelectric work function for a surface is 2.4 eV. Light of wavelength 6.8×10^{-7} m shines on the surface. Find the frequency of incident light and also the threshold frequency. Will there be photoelectric emission or not?

(Ans. $\nu = 4.41 \times 10^{14}$ Hz, $\nu_0 = 5.82 \times 10^{14}$ Hz, no photoelectric emission occurs as $\nu < \nu_0$)

19. If the speed of photoelectrons is 10^4 ms^{-1} , what should be the frequency of incident radiation on the potassium metal? Work function of potassium = 2.3 eV. (Ans. 5.56×10^{14} Hz)

20. Which of the metal sodium and copper will be suitable for photoelectric cell using light of wavelength 4000 \AA ? The work functions of sodium and copper are respectively 2.0 eV and 4.0 eV. Take $h = 6.6 \times 10^{-34}$ Js, $c = 3.0 \times 10^8 \text{ ms}^{-1}$ and $1 \text{ eV} = 1.6 \times 10^{-19}$ J) (Ans. Sodium)

21. (i) The work function for the surface of aluminium is 4.2 eV. How much potential difference will be required to stop the emission of maximum energy electrons emitted by light of 2000 \AA wavelength?

(ii) What will be the wavelength of that incident light for which stopping potential will be zero? Given $h = 6.6 \times 10^{-34}$ Js, $c = 3 \times 10^8 \text{ ms}^{-1}$.

[Ans. (i) 1.9875 V (ii) 2946 \AA]

22. The wavelength of a photon is 1.4 \AA . It collides with an electron at rest. Its wavelength after collision is 2.0 \AA . Calculate the energy of the scattered electron.

(Ans. 4.26×10^{-16} J)

23. The work function, for a given photosensitive surface equals 2.5 eV. When light of frequency, ν , falls on this surface, the emitted photoelectrons are completely stopped by applying a retarding potential of 4.1 V. What is the value of ν ? [CBSE OD 07C]

(Ans. 1.6×10^{15} Hz)

24. When light of frequency 2.4×10^{15} Hz, falls on a photosensitive surface, the retarding potential needed to completely stop the emitted photo-

electrons, is found to be 6.8 V. What is the work function (in eV) of the given photosensitive surface?

[CBSE OD 07C]

(Ans. 3.1 eV)

25. Ultraviolet radiations of wavelength 800 \AA and 700 \AA , when allowed to fall on a photosensitive surface are found to liberate electrons with maximum kinetic energies of 2 eV and 4.1 eV respectively. Calculate the value of Planck's constant. [IIT 83]

(Ans. 6.27×10^{-34} Js)

26. Find the frequency of light, which ejects electrons from a metal surface fully stopped by a retarding potential of 3 V. The photoelectric effect begins in this metal at frequency of $6 \times 10^{14} \text{ s}^{-1}$. Find the work function of the metal. [Himachal 96]

(Ans. $13.25 \times 10^{14} \text{ s}^{-1}$, 2.48 eV)

27. When UV light of wavelength 300 nm is incident on a metal plate, a negative potential of 0.59 V is required to stop the emission of photoelectrons. Calculate the energy of the incident photon and the work function for the metal in eV. [ISCE 2000]

(Ans. 4.125 eV, 3.585 eV)

28. A metal has a work function of 2.0 eV and is illuminated by monochromatic light of wavelength 500 nm. Calculate

(i) the threshold wavelength.

(ii) the maximum energy of photoelectrons.

(iii) the stopping potential.

[ISCE 94]

[Ans. (i) 6187.5 \AA , (ii) 0.475 eV, (iii) 0.475 V]

HINTS

1. Here $\lambda = 6600 \text{ \AA} = 6600 \times 10^{-10} \text{ m}$

Energy of photon

$$= \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6600 \times 10^{-10}} = 3 \times 10^{-19} \text{ J.}$$

Momentum of photon,

$$p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{6600 \times 10^{-10}} = 10^{-27} \text{ kg ms}^{-1}.$$

2. Frequency,

$$\nu = \frac{E}{h} = \frac{3.3 \times 10^{-20}}{6.6 \times 10^{-34}} \text{ Hz} = 0.5 \times 10^{14} \text{ Hz.}$$

3. Here $\lambda = 4000 \text{ \AA} = 4000 \times 10^{-10} \text{ m}$

$$\text{Frequency, } \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{4000 \times 10^{-10}} = 7.5 \times 10^{14} \text{ Hz.}$$

$$\text{Energy, } E = h\nu = 6.6 \times 10^{-34} \times 7.5 \times 10^{14} \\ = 4.95 \times 10^{-19} \text{ J.}$$

$$4. E_{\max} = \frac{hc}{\lambda_{\min}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.66 \times 10^{-10}} \\ = 3.0 \times 10^{-15} \text{ J.}$$

5. Energy of a photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} = 3.97 \times 10^{-19} \text{ J}$$

Energy emitted by 25 W source per minute
 $= 25 \times 60 = 1500 \text{ J}$

Number of photons emitted per minute

$$n = \frac{1500}{3.97 \times 10^{-19}} = 3.78 \times 10^{21}.$$

$$6. E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1} \\ = \frac{6.63 \times 3 \times 10^{-26}}{1.6 \times 10^{-19}} \text{ eV} = 1.243 \times 10^{-6} \text{ eV.}$$

$$7. E = 0.5 \text{ keV} = 0.5 \times 10^3 \text{ eV} = 0.5 \times 10^3 \times 1.6 \times 10^{-19} \text{ J} \\ = 8.0 \times 10^{-17} \text{ J}$$

$$v = \frac{E}{h} = \frac{8.0 \times 10^{-17}}{6.62 \times 10^{-34}} = 1.2 \times 10^{17} \text{ Hz}$$

8. If n is the number of photons emitted per second, then

$$n = \frac{P}{hv} = \frac{3.0 \times 10^{-3}}{6.6 \times 10^{-34} \times 5.0 \times 10^{14}} \\ = 9.1 \times 10^{15} \text{ photons/second.}$$

9. Energy of a photon,

$$E = \frac{200 \text{ Js}^{-1}}{4 \times 10^{20} \text{ s}^{-1}} = 5 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5 \times 10^{-19}} \\ = 4.0 \times 10^{-7} \text{ m} = 400 \text{ nm.}$$

$$10. E = hv_0 = 6.6 \times 10^{-34} \times 4.6 \times 10^{14} \text{ J} \\ = \frac{6.6 \times 4.6 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 1.9 \text{ eV.}$$

$$11. W_0 = hv_{\min} = \frac{hc}{\lambda_{\max}}$$

$$\therefore \lambda_{\max} = \frac{hc}{W_0} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3 \times 1.6 \times 10^{-19}} \\ = 4137.5 \times 10^{-10} \text{ m} = 4137.5 \text{ \AA}$$

$$12. (i) v_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14} \text{ Hz}$$

$$(ii) W_0 = hv_0 = 6.62 \times 10^{-34} \times 5 \times 10^{14} \text{ J} \\ = \frac{6.62 \times 5 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 2.07 \text{ eV.}$$

$$13. v_0 = \frac{W_0}{h} = \frac{0.1 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} \text{ Hz} \\ = 2.4 \times 10^{13} \text{ Hz}$$

14. Energy of a photon of orange light is

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{68 \times 10^{-8}} \text{ J} \\ = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{68 \times 10^{-8} \times 1.6 \times 10^{-19}} \text{ eV} = 1.83 \text{ eV}$$

As the energy of a photon of orange light is less than the work function of sodium (2.3 eV), so sodium does not show photoelectric emission with orange light.

15. (i) No. of photons required
 $= \text{No. of photoelectrons ejected}$

$$\text{or } n = \frac{q}{e} = \frac{11.2 \times 10^{-9}}{1.6 \times 10^{-19}} = 7 \times 10^{10}.$$

$$(ii) W = hv_0 = 6.6 \times 10^{-34} \times 4.5 \times 10^{14} \\ = 2.97 \times 10^{-19} \text{ J} \\ = \frac{2.97 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.856 \text{ eV.}$$

$$16. K_{\max} = \frac{hc}{\lambda} - W_0 \\ = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10} \times 1.6 \times 10^{-19}} - 2.01 \\ = 2.4825 - 2.01 = 0.4725 \text{ eV.}$$

$$17. K_{\max} = \frac{hc}{\lambda} - W_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6.2 \times 10^{-6} \times 1.6 \times 10^{-19}} - 0.1 \\ = 0.2 - 0.1 = 0.1 \text{ eV.}$$

$$18. v = \frac{c}{\lambda} = \frac{3 \times 10^8}{6.8 \times 10^{-7}} = 4.41 \times 10^{14} \text{ Hz}$$

$$v_0 = \frac{W}{h} = \frac{2.4 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} \\ = 5.82 \times 10^{14} \text{ Hz}$$

$$19. hv = \frac{1}{2} mv^2 + W_0 \\ = \frac{1}{2} \times 9.1 \times 10^{-31} \times (10^4)^2 + 2.3 \times 1.6 \times 10^{-19} \\ = 4.55 \times 10^{-23} + 3.68 \times 10^{-19} \\ = 3.680455 \times 10^{-19} \text{ J} \\ v = \frac{3.680455 \times 10^{-19}}{h} = \frac{3.680455 \times 10^{-19}}{6.62 \times 10^{-34}} \\ = 5.56 \times 10^{14} \text{ Hz.}$$

20. For sodium,

$$\lambda_0 = \frac{hc}{W_0} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.0 \times 1.6 \times 10^{-19}}$$

$$= 6.188 \times 10^7 \text{ m} = 6188 \text{ \AA}$$

For copper,

$$\lambda_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.0 \times 1.6 \times 10^{-19}}$$

$$= 3.094 \times 10^7 \text{ m} = 3094 \text{ \AA}$$

The longest wavelength that can eject electrons from Na is 6188 Å and that from Cu is 3094 Å. Hence for light of wavelength 4000 Å, sodium is suitable.

21. (i) Here
- $W_0 = 4.2 \text{ eV} = 4.2 \times 1.6 \times 10^{-19} \text{ J}$
- ,
-
- $\lambda = 2000 \text{ \AA} = 2000 \times 10^{-10} \text{ m}$
- ,
- $V_0 = ?$

The maximum K.E. of the emitted photoelectron,

$$K_{\max} = \frac{hc}{\lambda} - W_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2000 \times 10^{-10}} - 4.2 \times 1.6 \times 10^{-19}$$

$$= 3.18 \times 10^{-19} \text{ J}$$

Stopping potential,

$$V_0 = \frac{K_{\max}}{e}$$

$$= \frac{3.18 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.9875 \text{ V.}$$

- (ii) For threshold wavelength
- λ_0
- ,
- $K_{\max} = 0$
- . Hence

$$\lambda_0 = \frac{hc}{W_0} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.2 \times 1.6 \times 10^{-19}}$$

$$= 2.946 \times 10^{-7} \text{ m} = 2946 \text{ \AA}$$

22. $K = hc \left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right]$

$$= 6.63 \times 10^{-34} \times 3 \times 10^8 \left[\frac{1}{1.4 \times 10^{-10}} - \frac{1}{2 \times 10^{-10}} \right]$$

$$= 4.26 \times 10^{-16} \text{ J.}$$

- 23.
- $h\nu = eV_0 + W_0 = 4.1 \text{ eV} + 2.5 \text{ eV} = 6.6 \text{ eV}$

$$\nu = \frac{6.6 \text{ eV}}{h}$$

$$= \frac{6.6 \times 1.6 \times 10^{-19} \text{ J}}{6.6 \times 10^{-34} \text{ Js}}$$

$$= 1.6 \times 10^{15} \text{ Hz.}$$

24. $W_0 = h\nu - eV_0$

$$= \frac{6.6 \times 10^{-34} \times 2.4 \times 10^{15}}{1.6 \times 10^{-19}} \text{ eV} - 6.8 \text{ eV}$$

$$= 9.9 - 6.8 = 3.1 \text{ eV.}$$

25. Proceed as in Example 21 on page 11.12.

26. $h\nu = h\nu_0 + eV_0$ or $\nu = \nu_0 + \frac{eV_0}{h}$

$$\text{or } \nu = 6 \times 10^{14} + \frac{1.6 \times 10^{-19} \times 3}{6.62 \times 10^{-34}}$$

$$= 6 \times 10^{14} + 7.25 \times 10^{14}$$

$$= 13.25 \times 10^{14} \text{ s}^{-1}.$$

$$W_0 = h\nu_0 = 6.62 \times 10^{-34} \times 6 \times 10^{14}$$

$$= 3.97 \times 10^{-19} \text{ J} = 2.48 \text{ eV.}$$

27. Energy of incident photon,

$$h\nu = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}} \text{ J}$$

$$= \frac{6.6 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 4.125 \text{ eV.}$$

$$K_{\max} = eV_0 = 0.59 \text{ eV}$$

$$\therefore W_0 = h\nu - K_{\max} = 4.125 - 0.59$$

$$= 3.585 \text{ eV.}$$

28. (i) $W_0 = h\nu_0 = \frac{hc}{\lambda_0}$

$$\therefore \lambda_0 = \frac{hc}{W_0} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.0 \times 1.6 \times 10^{-19}}$$

$$= 6.1875 \times 10^{-7} \text{ m} = 6187.5 \text{ \AA}$$

(ii) $K_{\max} = \frac{hc}{\lambda} - W_0$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9} \times 1.6 \times 10^{-19}} - 2.0$$

$$= 2.475 - 2.0 = 0.475 \text{ eV.}$$

(iii) $V_0 = \frac{K_{\max}}{e} = 0.475 \text{ V.}$

11.10 PHOTO-CELL

10. What is a photo-cell? Mention the different types of photo-cells.

Photo-cell. A photo-cell or a photoelectric cell is an arrangement which converts light energy into electric energy. It works on the principle of photoelectric emission. It measures the intensity of light by measuring the photoelectric current, when light is incident on a photosensitive surface. Sometimes, it is also called an *electric eye*.

Photo-cells are of three types :

1. **Photo-emissive cell.** It makes use of photo-emission from a photosensitive cathode. The electrons emitted are attracted by an anode.

- Photo-conductive cell.** It makes use of the fact that electrical conductivity of a photosensitive semiconductor (e.g., selenium) increases when exposed to light due to excitation of additional free charge carriers by the incident photons.
- Photo-voltaic cell.** It makes use of the fact that an emf is produced between two layers of different materials as a result of irradiation. This cell supplies a current without using an external battery and is used in exposure-meters of good cameras and in solar batteries.

We describe here the most commonly used photo-emissive cell.

11. Describe construction and working of a photo-emissive cell.

Photo-emissive cell. It works on the principle of photoelectric emission.

Construction. It consists of an evacuated glass or quartz tube which encloses two electrodes, as shown in Fig. 11.12(a). The cathode or emitter is a parabolic metal

plate coated with a layer of some photosensitive material like oxides of Na, Cs, Rb, etc. The anode is a thin rod of Pt or Ni which faces the cathode. It is also known as a collector. The two electrodes are connected externally to a high tension battery and a microammeter (μA).

In another variant of photo-emissive cell shown in Fig. 11.12(b), a thin layer of a photo-sensitive material is pasted inside the bulb. A portion of the bulb is left clear for the light to enter it.

Working. When light of frequency greater than the threshold frequency falls on the cathode, photoelectrons are emitted which are attracted by the collector. The circuit gets completed and a current starts flowing in the circuit. As the number of photoelectrons emitted is proportional to the intensity of incident light, the photoelectric current indicated by the micrometer gives a measure of the intensity of light. The photoelectric current produced is very feeble, so it is first amplified before it is used for some useful purpose.

11.11 APPLICATIONS OF PHOTO-CELLS

12. Describe some important applications of photo-cells in daily life.

Applications of photocells in daily life. A photo-cell converts a change in intensity of radiation into a change in photo-current. This current can be used to operate different control systems and in light measuring devices as described below.

- In cinematography.** Photo-cells are used for the reproduction of sound. Audio signals are converted into electrical waves, which are then converted into light waves. The light signals are photographed on the film along with the action picture. With a reverse process, we get the picture and synchronised sound in the movie theatre.
- In counting devices.** A photo-cell is connected to a counter. When a person interrupts the invisible ultraviolet light falling on the photo-cell, the photoelectric current stops and the counter advances by one digit.
- In burglar's alarm.** When a person approaching a doorway interrupts a beam of invisible ultraviolet light falling on a photo-cell, the sudden change in photoelectric current starts a motor which opens the door or rings an alarm.
- In fire alarm.** In fire alarm, a number of photo-cells are installed at suitable places in a building. In the event of breaking out of fire, light radiations fall on the photo-cell. Photoelectric

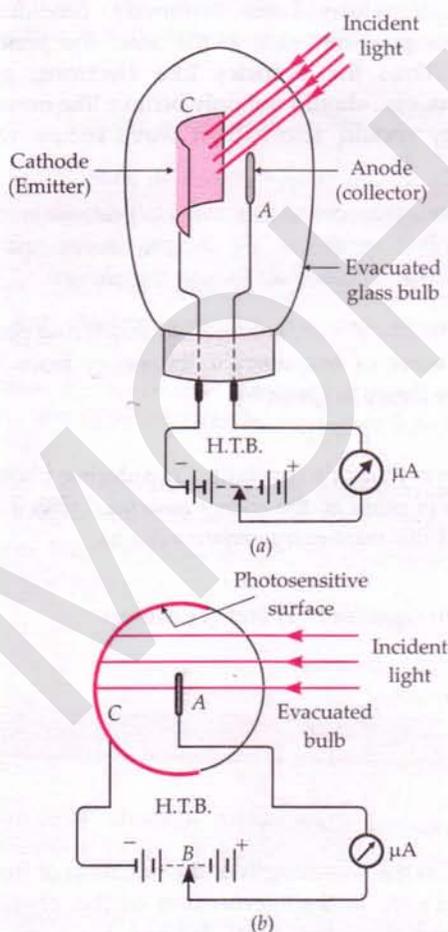


Fig. 11.12 Two variants of a photo-emissive cell.

current starts flowing through the circuit containing an electric bell or siren which starts operating as a warning signal.

5. In **photographic cameras**. As light meters in photographic cameras, photo-cells measure the intensity of light during photography.
6. In **automatic control of street light system**. Photo-cells inserted in the street light electric circuit, automatically switch on and switch off the street lighting system at dusk and dawn.
7. To operate controls in electronic devices such as television, computers, etc.
8. To study the temperature and spectra of stars.
9. To control the temperature of furnaces and chemical reactions.
10. In automatic control and checking of traffic signals and speed of automobile.
11. As **complexion metres**, by means of which we can compare the complexions of different persons.
12. In locating minor flaws or holes in metal sheets.
13. In the preparation of solar batteries.
14. In determining the opacity of solids and liquids.
15. In meteorology for recording daylight.

11.12 DUAL NATURE OF RADIATION

13. What do you mean by dual nature of radiation ?

Dual nature of radiation. The phenomena like interference, diffraction and polarisation, etc., can be satisfactorily explained only on the basis of wave nature of light. On the other hand, the phenomena like photoelectric effect, Compton effect, etc., can be explained only in terms of quantum theory of light, i.e., by assuming particle nature of light. This shows that *light radiation has dual nature, i.e., it sometimes behaves like a wave and sometimes as a particle.*

For Your Knowledge

- Radiation has dual nature : wave and particle. It is the nature of the experiment that decides whether a wave or particle description is best suited for understanding the experimental result. For example, when we see an object with our eye, both descriptions are necessary. The gathering and focussing mechanism of light by the eye-lens is well described by the wave picture. But the absorption of light by rods and cones (of the retina) can be described in the photon picture of light. Not a single experiment has been devised so far, which displays both wave and particle behaviours of radiation simultaneously.

11.13 DUAL NATURE OF MATTER : DE-BROGLIE WAVES

14. What considerations led de-Broglie to suggest that material particles can also have wave properties ? Derive de-Broglie wave equation for material particles.

Dual nature of matter : de-Broglie waves. In 1924, the French physicist *Louis Victor de-Broglie* (pronounced as de Broy) put forward the bold hypothesis that *material particles in motion should display wave-like properties*. His reasoning was based on the following two considerations :

1. The two physical quantities which govern all the forms of the physical universe are *mass* and *energy*.

The Einstein's mass-energy relationship :

$$E = mc^2,$$

shows that there is a complete equivalence between *matter* (mass) and *radiation* (energy). There must be a mutual symmetry between matter (mass) and radiation.

2. *Nature loves symmetry*. Since radiation has dual nature, therefore, from symmetry considerations, de-Broglie predicted that *matter must also possess dual nature*. Thus the particles like electrons, protons, neutrons, etc., should not only behave like mass points but they should also exhibit wave nature when in motion.

The waves associated with material particles in motion are called *matter or de Broglie waves* and their wavelength is called *de Broglie wavelength*.

de-Broglie's wave equation. Considering photon as an em wave of frequency ν , its energy from *Planck's quantum theory* is given by

$$E = h\nu \quad \dots(1)$$

where h is Planck's constant. Considering photon as a particle of mass m , the energy associated with it is given by *Einstein's mass-energy relationship* as

$$E = mc^2 \quad \dots(2)$$

From equations (1) and (2), we get

$$h\nu = mc^2$$

$$\text{or} \quad \frac{hc}{\lambda} = mc^2 \quad \left[\because \nu = \frac{c}{\lambda} \right]$$

$$\text{or} \quad \lambda = \frac{h}{mc} = \frac{h}{p}$$

where λ is the wavelength of the radiation of frequency ν and $p = mc$, is the momentum of the photon. The above equation has been derived for a photon of radiation. According to de Broglie's hypothesis, it

must be true for material particles like electrons, protons, neutrons, etc. Hence a particle of mass m moving with velocity v must be associated with a matter wave of wavelength λ given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \dots(3)$$

This is **de Broglie's wave equation** for material particles. It explains the dual nature of matter as it connects the wave characteristic ' λ ' with the particle characteristic ' p '.

From de-Broglie's equation, we find that

1. The wavelength of a moving particle is inversely proportional to its momentum,

$$\text{i.e.,} \quad \lambda \propto \frac{1}{p}$$

2. If $v = 0$, then $\lambda = \infty$. This implies that waves are associated with material particles only when they are in motion.

3. To be associated with a de Broglie wave, a particle need not have a charge. That is why, de-Broglie waves are also known as matter waves.

4. de-Broglie waves cannot be electromagnetic in nature because electromagnetic waves are only associated with accelerated charged particles.

For Your Knowledge

- The dualism of matter is inherent in the de Broglie equation : $\lambda = h/p$, because it contains a wave characteristic λ and a particle characteristic.
- The de Broglie wavelength is independent of the charge and nature of the material particle.
- de Broglie wavelength, $\lambda \propto 1/m$. This wavelength is significantly measurable (of the order of atomic-planes spacing in crystals) only in case of subatomic particles like electrons, protons, etc ; due to the smallness of their masses. But the de-Broglie wavelength of large moving objects is very small, quite beyond measurement, due to their large masses. That is why the *macroscopic objects in our daily life do not show wave-like properties.*
- The matter wave picture given by de Broglie elegantly incorporates the *Heisenberg's uncertainty* principle. According to this principle, *it is not possible to measure both the position and momentum of a subatomic particle at the same time exactly.* The product of the uncertainty in position (Δx) and the uncertainty in momentum (Δp) is never less than $h/4\pi$, i.e.,

$$\Delta x \times \Delta p \geq \frac{h}{4\pi}$$

- In order to describe the location of a particle, say electron, associated with a matter wave, *Max Born* suggested interpretation to the matter wave amplitude. According to this concept, the square of the amplitude (or intensity) of the matter wave at a point is related to the probability density of the particle (electron) at that point. So if the intensity of matter wave is large in a certain region, there is greater probability of the particle being found there.

- According to de Broglie relation, if an electron has a definite momentum p ($\Delta p = 0$), then it has a definite wavelength λ , $\lambda = h/p$. Such a wave of single wavelength extends all over space, as shown in Fig. 11.13(a). This implies that the electron is not localised in any finite region i.e., its position uncertainty is infinite ($\Delta x \rightarrow \infty$). This picture is consistent with the uncertainty principle.

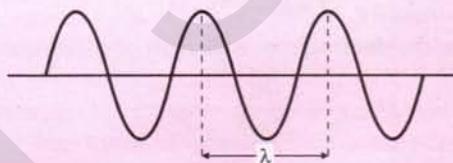


Fig. 11.13 (a) A matter wave associated with an electron of definite momentum has single wavelength and extends all over space.

- Generally, the matter wave associated with a particle does extend all over space. It is a wave packet that extends over some finite region of space. Such a wave packet does not have a single wavelength. It has a spread of wavelengths around some central wavelength, as shown in Fig. 11.13(b). Hence by de Broglie relation, there is a spread in momentum also. Consequently, the electron is associated with an uncertainty in position (Δx) and uncertainty in momentum (Δp). This wave packet picture of electron incorporating both de Broglie relation and Born's probability density concept is in complete agreement with the Heisenberg's uncertainty principle.

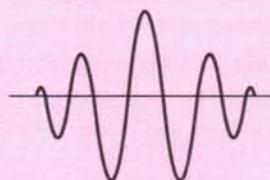


Fig. 11.13 (b) The wave packet description of an electron.

11.14 DE-BROGLIE WAVELENGTH OF AN ELECTRON

15. Deduce an expression for the de Broglie wavelength of an electron accelerated through a potential difference of V volts.

de-Broglie wavelength of an electron. Consider an electron of mass m and charge e . Let v be the final velocity attained by the electron when it is accelerated from rest through a potential difference of V volts. Then kinetic energy gained by the electron equals the work done on the electron by the electric field.

K.E. gained by the electron,

$$K = \frac{1}{2} mv^2 = \frac{p^2}{2m}$$

Work done on the electron = eV

$$\therefore K = \frac{p^2}{2m} = eV$$

or
$$p = \sqrt{2mK} = \sqrt{2meV}$$

Hence the de Broglie wavelength of the electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$$

Now $h = 6.63 \times 10^{-34}$ Js

$$m = 9.1 \times 10^{-31}$$
 kg

$$e = 1.6 \times 10^{-19}$$
 C

$$\therefore \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}$$

$$= \frac{12.3 \times 10^{-10}}{\sqrt{V}} \text{ m} = \frac{12.3}{\sqrt{V}} \text{ \AA}$$

For an accelerating potential of 120 V, we find $\lambda = 0.112$ nm. This wavelength is of the same order as the spacing between the atomic planes in crystals. This suggested that matter waves associated with electrons could be detected by electron diffraction experiments. *de Broglie* was awarded the Nobel Prize in Physics in 1929 for his discovery of the wave nature of electrons.

11.15 EXPERIMENTAL DEMONSTRATION OF WAVE NATURE OF ELECTRONS

16. Describe Davisson and Germer experiment to demonstrate the wave nature of electrons.

Davisson and Germer experiment. In 1927, *Davisson* and *Germer* designed an experiment to study the wave properties of electrons, which is shown in Fig. 11.14.

Here the electrons emitted by the hot filament of an electron gun are accelerated by applying a suitable potential difference V between the cathode and anode.

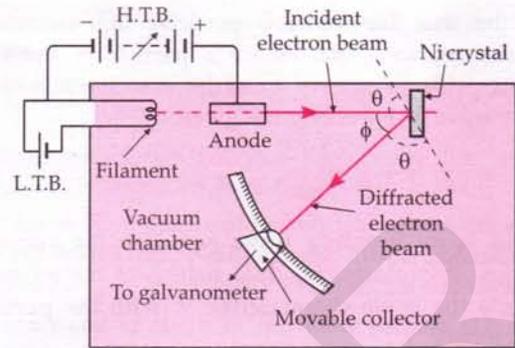


Fig. 11.14 Davisson and Germer electron diffraction apparatus.

The fine collimated beam of electrons from the electron gun is directed against the face of Ni crystal. The crystal is capable of rotation about an axis perpendicular to the plane of paper. The electrons, scattered in different directions by the atoms of Ni crystal, are received by a movable detector which is just an electron collector. Thus we measure scattered electron intensity as a function of the scattering angle ϕ , the angle between the incidence and the scattered electron beam. The experiment is repeated for different accelerating potentials V .

Figures 11.15 (a) to (e) show the results of Davisson and Germer experiment, when the accelerating voltage was varied from 44 V to 68 V. Clearly, there is a strong peak corresponding to a sharp diffraction maximum in the electron distribution at an accelerating voltage of 54 V and scattering angle 50° . The maximum of intensity obtained in a particular direction is due to constructive interference of electrons scattered from different layers of the regularly spaced atoms of the crystal.

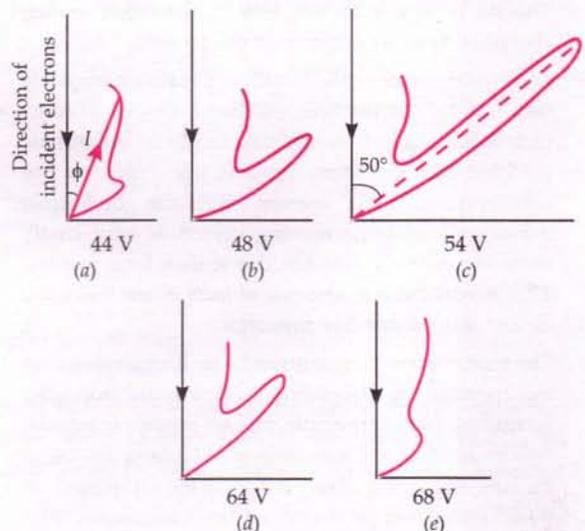


Fig. 11.15 Polar graphs showing the intensity of electrons as a function of scattering angle.

From Fig. 11.14, the glancing angle θ is given by

$$\theta + \phi + \theta = 180^\circ$$

$$\text{or } \theta = 90^\circ - \frac{1}{2} \phi = 90^\circ - 25^\circ = 65^\circ$$

The interatomic separation for Ni crystal is

$$d = 0.914 \text{ \AA}$$

For first order ($n=1$) diffraction maximum, the Bragg's law is

$$2d \sin \theta = \lambda$$

$$\therefore \lambda = 2 \times 0.914 \times \sin 65^\circ \\ = 2 \times 0.914 \times 0.906 \text{ \AA} = 1.65 \text{ \AA}$$

From de-Broglie hypothesis, the wavelength associated with an electron beam accelerated through 54 V must be

$$\lambda = \frac{h}{mv} = \frac{12.3}{\sqrt{V}} \text{ \AA} = \frac{12.3}{\sqrt{54}} \text{ \AA} \\ = 1.66 \text{ \AA}$$

The experimentally measured wavelength is close to that estimated from de-Broglie hypothesis. This proves the existence of de-Broglie waves.

17. Describe G.P. Thomson's experiment to demonstrate the wave nature of electrons.

G.P. Thomson's experiment. G.P. Thomson was able to obtain a diffraction pattern of an electron beam in 1928. The experimental set up is shown in Fig. 11.16. From an electron gun, a beam of electrons accelerated through a potential difference of 10 to 50 kV is incident on a thin platinum foil.

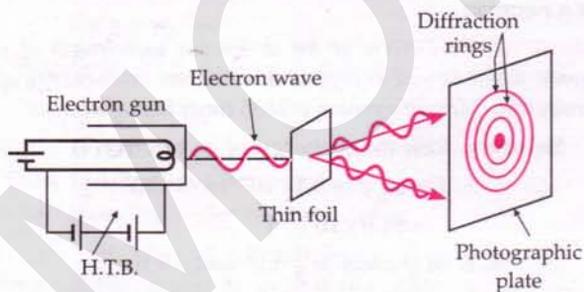


Fig. 11.16 Thomson's experiment to study electron diffraction.

The emergent beam is received on a photographic plate. The electron beam is diffracted at the spacings between the randomly oriented crystals of the thin foil. On the photographic plate, we get a circular diffraction pattern similar to Laue's X-ray diffraction pattern. This conclusively proves that the electron beams behave like waves.

11.16 ELECTRON MICROSCOPE

18. Briefly describe the working principle of an electron microscope.

Electron microscope. Electron microscope is an important application of de Broglie waves designed to study very minute objects like viruses, microbes and the crystal structure of the solids. It was first designed by Ernst Ruska in 1930.

Principle. Its working is based on the following facts :

1. Like light radiations, electron beams behave as waves but with much smaller wavelengths.
2. By using electric and magnetic fields, electron beams can be focussed just as ordinary light beams are focussed by glass lenses.

Theory. The de Broglie wavelength associated with an electron accelerated through a p.d. of V volts is given by

$$\lambda = \frac{12.3}{\sqrt{V}} \text{ \AA}$$

The magnifying power of a microscope is inversely related with the wavelength of radiation used. With an optical microscope, a magnification of ≈ 1500 is possible. This limit is due to the large wavelength of visible radiation. However, by selecting a suitable value of p.d. V , one can have an electron beam of as small wavelength as desired. That is why an electron microscope can have a very high magnification of $\approx 100,000$. In an electron microscope, the electrons are focussed with the help of electric and magnetic lenses.

Examples based on de-Broglie Waves

Formulae Used

$$1. \text{ Kinetic energy, } K = \frac{1}{2} mv^2 = \frac{p^2}{2m}$$

$$\therefore \text{ Momentum, } p = \sqrt{2mK}$$

$$2. \text{ de-Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

3. de-Broglie wavelength of an electron beam accelerated through a potential difference of V volts is

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{1.23}{\sqrt{V}} \text{ nm}$$

4. Bragg's equation for crystal diffraction is $2d \sin \theta = n\lambda$, n is order of the spectrum.

Units Used

Wavelength λ is in metre, velocity v in ms^{-1} , momentum p in kgms^{-1} , potential difference V in volt.

Example 23. What is the de Broglie wavelength associated with (i) an electron moving with a speed of $5.4 \times 10^6 \text{ ms}^{-1}$ and (ii) a ball of mass 1.50 g travelling at 30.0 m/s ?

[NCERT]

Solution. (i) $\lambda_e = \frac{h}{m_e v_e} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 5.4 \times 10^6}$
 $= 0.135 \times 10^{-9} \text{ m} = \mathbf{0.135 \text{ nm.}}$

(ii) $\lambda_b = \frac{h}{m_b v_b} = \frac{6.63 \times 10^{-34}}{0.150 \times 30.0} = \mathbf{1.47 \times 10^{-34} \text{ m.}}$

Example 24. What is the de Broglie wavelength associated with an electron, accelerated through a potential difference of 100 volts ?

[NCERT ; CBSE 06]

Solution. Accelerating potential, $V = 100 \text{ V}$

The de Broglie wavelength is

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm} = \frac{1.227}{\sqrt{100}} \text{ nm} \approx \mathbf{0.123 \text{ nm.}}$$

This wavelength is of the order of X-ray wavelengths.

Example 25. Calculate momentum of a photon associated with a radiation of frequency $5 \times 10^{13} \text{ Hz}$. Given $h = 6.6 \times 10^{-34} \text{ Js}$ and $c = 3 \times 10^8 \text{ ms}^{-1}$. [Himachal 2000C]

Solution. $p = \frac{h}{\lambda} = \frac{hv}{c} = \frac{6.6 \times 10^{-34} \times 5 \times 10^{13}}{3 \times 10^8}$
 $= \mathbf{1.1 \times 10^{-28} \text{ kg ms}^{-1}.}$

Example 26. The kinetic energy of the electron orbiting in the first excited state of hydrogen atom is 3.4 eV . Determine the de-Broglie wavelength associated with it. [CBSE F 15]

Solution. K.E. of the electron,

$$K = 3.4 \text{ eV} = 3.4 \times 1.6 \times 10^{-19} \text{ J}$$

de-Broglie wavelength associated with the electron,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}} \text{ m}$$

$$= \mathbf{6.63 \times 10^{-10} \text{ m.}}$$

Example 27. Calculate the de-Broglie wavelength associated with an α -particle accelerated through a potential difference of 200 V . Given $m_p = 1.67 \times 10^{-27} \text{ kg}$.

Solution. Mass of α -particle,

$$m = 4 m_p = 4 \times 1.67 \times 10^{-27} \text{ kg}$$

Charge on α -particle = $2e$

If the α -particle acquires velocity v , then

$$qV = \frac{1}{2} mv^2 \quad \text{or} \quad 2 eV = \frac{1}{2} m^2 v^2$$

$$\therefore mv = \sqrt{4 meV}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{4 meV}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{4 \times 4 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times 200}} \text{ m}$$

$$= \frac{6.6 \times 10^{-34}}{10^{-23} \sqrt{16 \times 1.67 \times 1.6 \times 200}} = \frac{6.6 \times 10^{-11}}{92.47}$$

$$= 0.07138 \times 10^{-11} \text{ m} = \mathbf{0.007138 \text{ \AA.}}$$

Example 28. A particle is moving three times as fast as an electron. The ratio of the de Broglie wavelength of the particle to that of the electron is 1.813×10^{-4} . Calculate the particle's mass and identify the particle. Mass of electron = $9.11 \times 10^{-31} \text{ kg}$. [NCERT]

Solution. de Broglie wavelengths of the particle and the electron are

$$\lambda_p = \frac{h}{m_p v_p} \quad \text{and} \quad \lambda_e = \frac{h}{m_e v_e}$$

$$\therefore \frac{\lambda_p}{\lambda_e} = \frac{m_e v_e}{m_p v_p} \quad \text{or} \quad \frac{m_p}{m_e} = \frac{\lambda_e}{\lambda_p} \cdot \frac{v_e}{v_p}$$

Given $v_p = 3 v_e$ and $\frac{\lambda_p}{\lambda_e} = 1.813 \times 10^{-4}$

$$\therefore \frac{m_p}{m_e} = \frac{1}{1.813 \times 10^{-4}} \times \frac{1}{3}$$

or $m_p = \frac{m_e}{3 \times 1.813 \times 10^{-4}} = \frac{9.11 \times 10^{-31}}{3 \times 1.813 \times 10^{-4}} \text{ kg}$

$$= \mathbf{1.675 \times 10^{-27} \text{ kg}}$$

Thus the particle, with this mass, could be a proton or a neutron.

Example 29. Determine the de-Broglie wavelength of a proton whose kinetic energy is equal to rest mass energy of an electron. Mass of a proton is 1836 times that of electron.

Solution. Rest mass energy of an electron is

$$E = m_0 c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \text{ J}$$

$$= 81.9 \times 10^{-15} \text{ J}$$

$$\therefore \text{K.E. of proton} = \frac{1}{2} mv^2 = 81.9 \times 10^{-15}$$

or $\frac{1}{2} \times 1836 \times 9.1 \times 10^{-31} \times v^2 = 81.9 \times 10^{-15}$

or $v = \left[\frac{2 \times 81.9 \times 10^{16}}{1836 \times 9.1} \right]^{1/2} \text{ ms}^{-1}$

\therefore de-Broglie wavelength, $\lambda = \frac{h}{mv}$

$$= \frac{6.63 \times 10^{-34}}{1836 \times 9.1 \times 10^{-31}} \times \left[\frac{1836 \times 9.1}{2 \times 81.9 \times 10^{16}} \right]^{1/2}$$

$$= \mathbf{4 \times 10^{-14} \text{ m.}}$$

Example 30. Find the de-Broglie wavelength associated with an electron moving with a velocity $0.5c$ and rest mass $= 9.1 \times 10^{-31}$ kg.

Solution. Here $m_0 = 9.1 \times 10^{-31}$ kg,
 $v = 0.5c = 0.5 \times 3 \times 10^8 = 1.5 \times 10^8 \text{ ms}^{-1}$

Mass of electron in motion (relativistic mass) is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{(0.5c)^2}{c^2}}}$$

$$= \frac{m_0}{\sqrt{1 - 0.25}} = \frac{m_0}{\sqrt{0.75}}$$

$$\therefore \lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34} \times \sqrt{0.75}}{m_0 v}$$

$$= \frac{6.6 \times 10^{-34} \times \sqrt{0.75}}{9.1 \times 10^{-31} \times 1.5 \times 10^8} = 4.2 \times 10^{-12} \text{ m.}$$

Example 31. Find the typical de-Broglie wavelength of an electron in a metal at 27°C and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m. [NCERT]

Solution. Mass of an electron is

$$m = 9.11 \times 10^{-31} \text{ kg and } T = 27 + 273 = 300 \text{ K}$$

\therefore de-Broglie wavelength of electrons is

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}} \text{ m}$$

$$= \frac{6.63 \times 10^{-8}}{\sqrt{3 \times 9.11 \times 1.38 \times 3}} = \frac{6.63 \times 10^{-8}}{10.64}$$

$$= 6.2 \times 10^{-9} \text{ m}$$

Mean separation between two electrons in a metal is

$$r = 2 \times 10^{-10} \text{ m} \quad \therefore \frac{\lambda}{r} = \frac{6.2 \times 10^{-9}}{2 \times 10^{-10}} = 31$$

Thus the de-Broglie wavelength is much greater than the given inter-electron separation.

Example 32. The equivalent wavelength of a moving electron has the same value as that of a photon having an energy of 6×10^{-17} J. Calculate the momentum of the electron. [CBSE SP 15]

Solution. Energy of a photon, $E = hv = \frac{hc}{\lambda}$

Wavelength of the photon, $\lambda = \frac{hc}{E}$

Momentum of the moving electron,

$$p = \frac{h}{\lambda} = \frac{hE}{hc} = \frac{E}{c} = \frac{6 \times 10^{-17}}{3 \times 10^8} = 2 \times 10^{-25} \text{ kgms}^{-1}.$$

Example 33. The extent of localisation of a particle is determined roughly by its de Broglie wavelength. If an electron is localized within the nucleus (of size about 10^{-14} m) of an atom, what is its energy? Compare this energy with the typical binding energies (of the order of a few MeV) in a nucleus, and hence argue why electrons cannot reside in a nucleus. [NCERT]

Solution. As the electron is localised within the nucleus (of size about 10^{-14} m) of an atom, so

$$\lambda = 10^{-14} \text{ m}$$

$$\therefore p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-14}} = 6.63 \times 10^{-20} \text{ kg ms}^{-1}$$

The relativistic formula for the energy of an electron is

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

Neglecting the rest-mass energy (second) term, we get

$$E = pc = 6.63 \times 10^{-20} \times 3 \times 10^8 \text{ J}$$

$$= \frac{6.63 \times 3 \times 10^{-12}}{1.6 \times 10^{-13}} \text{ MeV} = 124.3 \text{ MeV.}$$

This is too much large compared to the binding energy that Coulomb force can provide within the nucleus. Therefore, electrons localised within a nucleus are far too energetic to stay bound within. That is why electrons do not reside in a nucleus.

Example 34. Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27°C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions. [NCERT]

Solution. Mass of the atom is given by

$$m = \frac{\text{Atomic wt. of He}}{\text{Avogadro's number}}$$

$$= \frac{4}{6 \times 10^{23}} \text{ g} = \frac{4 \times 10^{-3}}{6 \times 10^{23}} \text{ kg} = \frac{2}{3} \times 10^{-26} \text{ kg}$$

$$T = 27 + 273 = 300 \text{ K}$$

Average K.E. of a He atom at absolute temperature T is

$$\frac{1}{2} mv^2 = \frac{3}{2} kT$$

$$\therefore m^2 v^2 = 3mkT \quad \text{or} \quad p^2 = 3mkT$$

$$p = \sqrt{3mkT}$$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times \frac{2}{3} \times 10^{-26} \times 1.38 \times 10^{-23} \times 300}}$$

$$= 0.73 \times 10^{-10} \text{ m.}$$

Kinetic gas equation for one mole of a gas can be written as

$$PV = RT$$

or $PV = kNT$

$$\left[\because k = \frac{R}{N} \right]$$

or $\frac{V}{N} = \frac{kT}{P}$

\therefore Mean separation,

$$r = \left[\frac{\text{Molar Volume}}{\text{Avogadro's Number}} \right]^{1/3} = \left[\frac{kT}{P} \right]^{1/3}$$

Given $T = 300 \text{ K}$

$$P = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$$

$$k = 1.38 \times 10^{-23} \text{ J mol}^{-1} \text{ K}^{-1}$$

Hence $r = \left[\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5} \right]^{1/3} \text{ m}$

$$= \left[\frac{138 \times 30}{101} \right]^{1/3} \times 10^{-9} \text{ m}$$

$$= 3.4 \times 10^{-9} \text{ m}$$

We find that $r > \lambda$, i.e., the wave packets associated with He atoms do not overlap and hence He atoms can be distinctly seen.

Problems For Practice

- Calculate the de Broglie wavelength of a proton of momentum $2.55 \times 10^{-22} \text{ kg ms}^{-1}$. (Ans. 0.026 \AA)
- An electron is accelerated through a potential difference of 64 volts. What is the de-Broglie wavelength associated with it? To which part of the electromagnetic spectrum does this value of wavelength correspond? [CBSE D 10]
- What potential difference must be applied to an electron microscope to produce an electron beam of wavelength 0.41 \AA ? (Ans. 900 V)
- Calculate the de Broglie wavelength of an electron of kinetic energy 100 eV . Given $m = 9.1 \times 10^{-31} \text{ kg}$, $e = 1.6 \times 10^{-19} \text{ C}$, and $h = 6.62 \times 10^{-34} \text{ Js}$. [Himachal 04; NCERT] (Ans. 1.227 \AA)
- For what kinetic energy of a proton, will the associated de Broglie wavelength be 16.5 nm ? Mass of proton = $1.675 \times 10^{-27} \text{ kg}$, $h = 6.63 \times 10^{-34} \text{ Js}$. [CBSE OD 08C] (Ans. $5.04 \times 10^{-25} \text{ J}$)
- An electron is revolving around the nucleus with a constant speed of $2.5 \times 10^8 \text{ m/s}$. Find the de-Broglie wavelength associated with it. [CBSE OD 14C] (Ans. 2.9 pm)

- An electron and a photon each have a wavelength 2 nm . Find
 - their momenta,
 - the energy of the photon and
 - the kinetic energy of electron. [CBSE D 11]

HINTS

$$2. \lambda = \frac{12.27}{\sqrt{V}} \text{ \AA} = \frac{12.27}{\sqrt{64}} \text{ \AA} = 1.53 \text{ \AA}$$

This wavelength corresponds to X-rays.

- When a potential difference V is applied to an electron microscope, de-Broglie wavelength,

$$\lambda = \frac{12.3}{\sqrt{V}} \text{ \AA} \quad \therefore 0.41 = \frac{12.3}{\sqrt{V}}$$

or $V = \left(\frac{12.3}{0.41} \right)^2 = 900 \text{ V}$

$$4. K = 100 \text{ eV} = 1.6 \times 10^{-17} \text{ J}$$

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-17}}}$$

$$= 1.227 \times 10^{-10} \text{ m} = 1.227 \text{ \AA}$$

$$5. K = \frac{1}{2} mv^2 = \frac{1}{2m} (mv)^2 = \frac{1}{2m} \cdot \frac{h^2}{\lambda^2}$$

$$= \frac{(6.63 \times 10^{-34})^2}{2 \times 1.675 \times 10^{-27} \times (16.5 \times 10^{-9})^2}$$

$$= 5.04 \times 10^{-25} \text{ J}$$

$$6. \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.5 \times 10^8}$$

$$= 2.9 \times 10^{-12} \text{ m} = 2.9 \text{ pm}$$

$$7. \text{ Here, } \lambda = 2 \text{ nm} = 2 \times 10^{-9} \text{ m}$$

(a) Both electron and photon have the same momentum.

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2 \times 10^{-9}}$$

$$= 3.315 \times 10^{-25} \text{ kg ms}^{-1}$$

(b) Energy of a photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-9}} \text{ J}$$

$$= \frac{9.945 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV} = 6.22 \times 10^2 \text{ eV}$$

(c) Kinetic energy of the electron,

$$K = \frac{p^2}{2m} = \frac{(3.315 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} \text{ J}$$

$$= 0.6038 \times 10^{-19} \text{ J} = 0.377 \text{ eV}$$

VERY SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. Define photoelectric work function. How is it related to threshold frequency ?

[Haryana 04 ; ISCE 90]

Solution. The minimum amount of radiant energy needed to pull an electron (without imparting it any kinetic energy) from a metallic surface is called work function of the metal. The relation between work function W_0 and threshold frequency ν_0 is $W_0 = h\nu_0$.

Problem 2. How will you justify that the rest mass of photons is zero ?

Solution. The mass of a body moving with speed v is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{Rest mass, } m_0 = m \sqrt{1 - \frac{v^2}{c^2}}$$

For a photon, $v = c$, therefore,

$$m_0 = m \sqrt{1 - \frac{c^2}{c^2}} = \text{zero.}$$

Problem 3. Do all the photons have same mass ? If not, why ?

Solution. Mass of a photon = $\frac{E}{c^2} = \frac{h\nu}{c^2}$

Different radiations have different frequencies. So their photons will have different masses.

Problem 4. Which photon is more energetic : A red one or a violet one ?

[Himachal 93 ; Haryana 02]

Solution. Violet photon has more energy, because energy of a photon,

$$E = h\nu \quad \text{and} \quad \nu_{\text{violet}} > \nu_{\text{red}}$$

Problem 5. If the wavelength of an electromagnetic radiation is doubled, what will happen to the energy of photons ?

[CBSE D 93]

Solution. Energy of a photon,

$$E = h\nu = \frac{hc}{\lambda} \quad \text{i.e., } E \propto \frac{1}{\lambda}$$

Clearly, energy of photon reduces to one-half when the wavelength of radiation is doubled.

Problem 6. What happens to the wavelength of a photon after it collides with an electron ?

Solution. A photon transfers a part of its energy to the colliding electron, so its energy decreases and consequently wavelength increases.

Problem 7. Why are alkali metals most suited as photo-sensitive metals ?

Solution. Alkali metals have low work function. Even visible radiation can eject out electrons from them. So alkali metals are most suitable photo-sensitive metals.

Problem 8. Does each incident photon essentially eject an electron ?

Solution. No, it may be absorbed in some other manner. Only about 1% of incident photons are capable of ejecting out electrons.

Problem 9. Is photoelectric emission possible at all frequencies ? Give reason for your answer.

[CBSE OD 90]

Solution. No. Photoelectric emission is possible only if the energy of the incident photon is greater than the work function ($W_0 = h\nu_0$) of the metal. Hence, the frequency ν of the incident radiation must be greater than the threshold frequency ν_0 .

Problem 10. Work function of aluminium is 4.2 eV. If two photons each of energy 2.5 eV are incident on its surface, will the emission of electrons take place ? Justify your answer.

[CBSE F 94]

Solution. No. Energy of a single photon must be greater than the work of the metal for the emission of a photoelectron.

Problem 11. Out of microwaves, ultraviolet rays and infra-red rays, which radiations will be most effective for emission of electrons from a metallic surface ?

[CBSE F 94]

Solution. Ultraviolet rays are most effective for photoelectric emission because they have highest frequency and hence most energetic.

Problem 12. Can X-rays cause photoelectric effect ?

Solution. Yes. X-rays can cause photoelectric effect in sodium, zinc and copper.

Problem 13. Two metals A and B have work functions 4 eV and 10 eV respectively. Which metal has higher threshold wavelength ?

[CBSE OD 04]

$$\text{Solution. } W_0 = h\nu_0 = \frac{hc}{\lambda_0} \quad \text{i.e., } \lambda_0 \propto \frac{1}{W_0}$$

So metal A with lower work function has higher threshold wavelength.

Problem 14. Ultraviolet light is incident on two photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$). In which case will the kinetic energy of the emitted electrons be greater ? Why ?

[CBSE OD 05]

$$\text{Solution. K.E. of a photoelectron} = h\nu - W_0$$

Hence the kinetic energy of the emitted electrons will be greater for the photo-sensitive material having smaller work function W_2 .

Problem 15. Does the 'stopping potential' in photoelectric emission depend upon

- (i) the intensity of the incident radiation in a photocell ?
 (ii) the frequency of the incident radiation ?

[CBSE D 05]

Solution. (i) No, the stopping potential does not depend on the intensity of the incident radiation.

(ii) Yes, the stopping potential depends on the frequency of incident radiation. Above the threshold frequency,

$$V_0 \propto \nu.$$

Problem 16. Two beams, one of red light and the other of blue light, of the same intensity are incident on a metallic surface to emit photoelectrons. Which one of the two beams emits electrons of greater kinetic energy ?

[CBSE D 04C]

Solution. Blue light emits electrons of greater kinetic energy because its frequency is higher than that of red light.

K.E. of a photoelectron \propto Frequency of incident radiation

Problem 17. Electrons are emitted from a photo-sensitive surface when it is illuminated by green light but electron emission does not take place by yellow light. Will the electrons be emitted when the surface is illuminated by (i) red light, and (ii) blue light ?

[CBSE D 05]

Solution. (i) No, electrons are not emitted by yellow light, because

$$\nu_{\text{red}} < \nu_{\text{yellow}}$$

(ii) Yes, electrons are emitted by blue light, because

$$\nu_{\text{blue}} > \nu_{\text{green}}$$

Problem 18. When a monochromatic yellow coloured light beam is incident on a given photosensitive surface, photoelectrons are not ejected, while the same surface gives photoelectrons when exposed to green coloured monochromatic beam. What will happen if the same photo-sensitive surface is exposed to (i) violet and (ii) red coloured, monochromatic beam of light ? Justify your answer.

[CBSE OD 2000C, 01C]

Solution. (i) Violet light has higher frequency than green light, so it can eject electrons from the photosensitive surface.

(ii) Red light has lower frequency than yellow light, it cannot eject electrons from the photo-sensitive surface.

Problem 19. How does the maximum kinetic energy of electrons emitted vary with the work function of the metal ?

[CBSE D 12]

Solution. The maximum kinetic energy of emitted electrons,

$$K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 = h\nu - W_0$$

Clearly, the larger the work function of the metal, lesser is the maximum K.E. of the photoelectrons.

Problem 20. A source of light is placed at a distance of 50 cm from a photo-cell and the cut-off potential is found to be V_0 . If the distance between the light source and photo-cell is made 25 cm, what will be the new cut-off potential ? Justify your answer. [CBSE D 01C]

Solution. The stopping potential is still V_0 . As the distance is decreased from 50 cm to 25 cm (i.e., the distance is halved), the intensity of light becomes four times the original intensity. But the stopping potential is independent of the intensity.

Problem 21. When monochromatic radiation of wavelength 2000 Å falls upon a nickel plate, the latter acquires a positive charge. The wavelength is increased and at 3400 Å, however intense the monochromatic radiation may be, effect is found to cease. Give reason.

Solution. Clearly, $\lambda_0 = 3400 \text{ \AA}$ is the threshold wavelength for nickel plate. When $\lambda \leq \lambda_0$, the incident photons have energy more than or equal to work function of Ni, so photoelectric emission occurs. When $\lambda > \lambda_0$, the energy of photons is less than the work function, so photoelectric emission does not occur.

Problem 22. What is the effect on the velocity of photoelectrons, if the wavelength of incident light is decreased ?

Solution. Kinetic energy of photoelectrons is given by Einstein's photoelectric equation,

$$\begin{aligned} K_{\text{max}} &= \frac{1}{2} m v_{\text{max}}^2 \\ &= h\nu - W_0 = \frac{hc}{\lambda} - W_0 \end{aligned}$$

$$\therefore v_{\text{max}}^2 \propto \frac{1}{\lambda}$$

$$\text{or } v_{\text{max}} \propto \frac{1}{\sqrt{\lambda}}$$

As the wavelength of incident light decreases, the velocity of photoelectrons increases.

Problem 23. It is difficult to eject out an electron from copper than sodium. Which of the two metals has greater work function and which has greater threshold wavelength ?

Solution. Since electron ejection is difficult from copper than sodium, so copper has greater work function than sodium.

Work function,

$$W_0 = hv_0 = \frac{hc}{\lambda_0} \quad \text{i.e.,} \quad \lambda_0 \propto \frac{1}{W_0}$$

As threshold wavelength is inversely related with work function, so sodium has higher threshold wavelength than copper.

Problem 24. The frequency (ν) of incident radiation is greater than threshold frequency (ν_0) in a photocell. How will the stopping potential vary if frequency (ν) is increased, keeping other factors constant? [CBSE D 02]

Solution. For frequency $\nu > \nu_0$, stopping potential $\propto \nu$. So the stopping potential increases, when the frequency ν is increased.

Problem 25. The stopping potential in an experiment on photoelectric effect is 1.5 V. What is the maximum kinetic energy of the photoelectrons emitted? [CBSE OD 09]

$$\text{Solution. } K_{\max} = eV_0 = 1.6 \times 10^{-19} \text{ C} \times 1.5 \text{ V} \\ = 2.4 \times 10^{-19} \text{ J or } 1.5 \text{ eV.}$$

Problem 26. The maximum kinetic energy of a photoelectron is 3 eV. What is its stopping potential? [CBSE OD 09]

$$\text{Solution. Stopping potential, } V_0 = \frac{K_{\max}}{e} = \frac{3\text{eV}}{e} = 3 \text{ V.}$$

Problem 27. State de-Broglie hypothesis. [CBSE D 12]

Solution. According to de-Broglie hypothesis, material particles in motion display wave-like properties.

Problem 28. What considerations led de-Broglie to suggest that material particles can also show wave property?

Solution. (i) de-Broglie concept of nature loves symmetry, and (ii) matter can be converted into energy and vice versa.

Problem 29. Are matter waves electromagnetic? Write de-Broglie equation. [Haryana 92]

Solution. No, matter waves are not electromagnetic. The de-Broglie equation is

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

Problem 30. Why are de-Broglie waves associated with a moving football not visible? [Haryana 07, 08; CBSE D 03]

Solution. Due to the large mass of a football, the de-Broglie wavelength ($\lambda = h/mv$) associated with a moving football is very small, quite beyond measurement. Hence its wave nature is not visible.

Problem 31. What inference was drawn from Davisson and Germer experiment regarding the nature of electrons? [ISCE 02]

Solution. This experiment confirmed the existence of electron waves.

Problem 32. The de-Broglie wavelength of a particle of kinetic energy K is λ . What would be the wavelength of the particle, if its kinetic energy were $K/4$? [ISCE 93]

Solution. de Broglie wavelength of a particle of mass m and kinetic energy K is

$$\lambda = \frac{h}{\sqrt{2mK}}$$

When the kinetic energy is $K/4$,

$$\lambda' = \frac{h}{\sqrt{2mK/4}} = \frac{2h}{\sqrt{2mK}} = 2\lambda.$$

Problem 33. The most probable kinetic energy of thermal neutrons at a temperature of T kelvin, may be taken as equal to kT , where k is Boltzmann constant. Taking the mass of a neutron and its associated de-Broglie wavelength as m and λ_B respectively, state the dependence of λ_B on m and T . [CBSE Sample Paper 11]

$$\text{Solution. } \lambda_B = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m \times kT}} \quad \therefore \lambda_B \propto \frac{1}{\sqrt{mT}}$$

Problem 34. Why is a photo-cell also called an electric eye?

Solution. Like an eye, a photo-cell can distinguish between a weak and an intense light. But a photocell gives a measure of light intensity in terms of photoelectric current. So it is also called an electric eye.

Problem 35. On what principle is an electron microscope based?

Solution. An electron microscope exploits the wave nature of an accelerated beam of electrons to provide high magnifying and resolving powers.

SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. (a) Describe briefly the experimentally observed features in the phenomenon of photoelectric effect.

(b) Discuss briefly how wave theory of light cannot explain these features. [CBSE OD 15]

Solution. (a) (i) For every metal there is a certain minimum frequency (threshold frequency) below which no photoelectrons are emitted, however high is the intensity of incident radiation.

(ii) Photoelectric current is directly proportional to the intensity of incident radiation.

- (iii) The photoelectric current becomes zero at a certain value of negative potential (stopping potential) applied at the anode.
- (iv) The value of stopping potential increases with the increase in the frequency of incident radiation.
- (v) The maximum K.E. of photoelectrons is directly proportional to the frequency of incident radiation.
- (vi) The maximum K.E. of photoelectron is independent of the incident radiation.
- (vii) The photoelectric emission is an instantaneous process.

(b) Wave theory of light fails to explain the following features of photoelectric effect :

- (i) The photoelectric emission is an instantaneous process.
- (ii) There exists a threshold frequency for every metal.
- (iii) The maximum K.E. of photoelectrons is independent of the intensity of incident radiation.

Problem 2. Write the basic features of the photon picture of electromagnetic radiation on which Einstein's photoelectric equation is based. [CBSE D 13, OD 13]

Or

Write three basic properties of photons which are used to obtain Einstein's photoelectric equation.

[CBSE OD 14C, 15]

Solution. The basic features of the photon picture of e.m. radiation are as follows :

- (i) Light is composed of discrete packets of energy called quanta or photons.
- (ii) Each photon carries an energy $E (= h\nu)$ and momentum $p (= h/\lambda)$, which depend on the frequency ν of the incident radiation and not on its intensity.
- (iii) Photoelectric emission from the metal surface occurs due to the absorption of a photon by an electron.

Problem 3. Write Einstein's photoelectric equation. State clearly how this equation is obtained using the photon picture of electromagnetic radiation. Write the three salient features observed in photoelectric effect which can be explained using this equation. [CBSE OD 10 ; D 12]

Or

What is photo-electric effect ? Write Einstein's photoelectric equation. Explain how it enables us to understand the

- (i) linear dependence, of the maximum kinetic energy of the emitted electrons, on the frequency of the incident radiation.

- (ii) existence of a threshold frequency for a given photoemitter.
- (iii) independence of the maximum energy of emitted photo-electrons from the intensity of incident light. [CBSE 01C, 04C]

Solution. Photo-electric effect is the phenomenon of emission of electrons from the surface of metals, when light radiations of suitable frequency fall on them.

In photoelectric effect, an electron of the metal surface absorbs a photon of energy $h\nu$. If this energy exceeds the work function of the metal, the electron is emitted with maximum K.E. By conservation of energy,

Energy of incident photon

= Maximum K.E. of photoelectron
+ Work function of metal

$$h\nu = \frac{1}{2}mv_{max}^2 + W_0$$

or

$$K_{max} = \frac{1}{2}mv_{max}^2 = h\nu - W_0$$

or

At threshold frequency ν_0 , no K.E. is given to the electron. So

$$h\nu_0 = W_0$$

Hence $K_{max} = h\nu - W_0 = h(\nu - \nu_0)$

This is Einstein's photoelectric equation.

- (i) Clearly, above the threshold frequency ν_0 , $K_{max} \propto \nu$ i.e., the maximum K.E. of the emitted electrons depends linearly on the frequency of incident radiation.
- (ii) When $\nu < \nu_0$, K_{max} becomes negative. The kinetic energy becomes negative which has no physical meaning. Hence there is no photoelectric emission below the threshold frequency ν_0 .
- (iii) It is obvious from the photo-electric equation that the maximum K.E. of photo-electrons does not depend on the intensity of incident light.

Problem 4. Write two characteristic features observed in photoelectric effect which support the photon picture of electromagnetic radiation.

Draw a graph between the frequency of incident radiation (ν) and the maximum kinetic energy of the electrons emitted from the surface of a photosensitive material. State clearly how this graph can be used to determine (i) Planck's constant and (ii) work function of the material. [CBSE F 12]

Solution. The following features observed in photoelectric effect support the photon picture of e.m. radiation :

- (i) The maximum kinetic energy of emitted electrons is independent of the intensity of incident radiation.

- (ii) For each photoemitter, there exists a threshold frequency (of incident light) below which no emission takes place.
- (iii) Photoelectric emission is an instantaneous process.

According to Einstein's photoelectric equation,

$$K_{\max} = h\nu - W_0$$

So the graph between K_{\max} and ν is a straight line as shown in Fig. 11.17

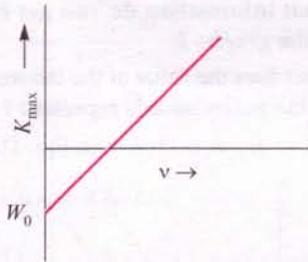


Fig. 11.17

(i) Slope of K_{\max} - ν graph

$$= \frac{\Delta K_{\max}}{\Delta \nu} = h$$

\therefore Slope of K_{\max} - ν graph gives the value of Planck's constant.

(ii) Intercept on the negative K_{\max} axis = W_0

\therefore Intercept on the negative K_{\max} axis gives the value of work function.

Problem 5. Which two main observations in photoelectricity led Einstein to suggest the photon theory for the interaction of light with the free electrons in a metal? Obtain an expression for the threshold frequency for photoelectric emission in terms of the work function of the metal. [CBSE Sample Paper 11]

Solution. For observations in photoelectricity, refer to the solution of Problem 4 above.

Energy of incident photon

$$= \text{Maximum K.E. of photoelectron} \\ + \text{Work function of metal}$$

$$\text{or } h\nu = \frac{1}{2}mv_{\max}^2 + W_0$$

At threshold frequency ν_0 , no K.E. is given to the photoelectron, so

$$h\nu_0 = W_0 \quad [K_{\max} = \frac{1}{2}mv_{\max}^2 = 0]$$

$$\text{or } \nu_0 = \frac{W_0}{h}$$

Problem 6. Define the terms (i) 'cut-off voltage' and (ii) 'threshold frequency' in relation to the phenomenon of photoelectric effect.

Using Einstein's photoelectric equation show how the cut-off voltage and threshold frequency for a given photosensitive material can be determined with the help of a suitable plot/graph. [CBSE OD 12]

Solution. (i) *Cut-off voltage or stopping potential.* The minimum negative potential given to the anode of a photocell for which the photoelectric current becomes zero is called cut-off voltage or stopping potential.

(ii) *Threshold frequency.* The minimum value of the frequency of incident radiation below which the photoelectric emission stops is called threshold frequency.

According to Einstein's photoelectric equation, the maximum K.E. of a photoelectron is given by

$$K_{\max} = h\nu - W_0$$

If V_0 is the stopping potential, then

$$K_{\max} = eV_0$$

$$\therefore eV_0 = h\nu - W_0$$

$$\text{or } V_0 = \left(\frac{h}{e}\right)\nu - \frac{W_0}{e} \quad \dots(1)$$

So the graph of V_0 versus ν is a straight line as shown in Fig. 11.18. We can read the value of threshold frequency ν_0 from the graph.

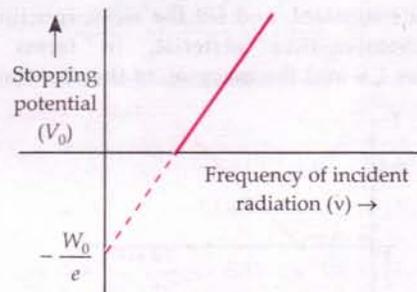


Fig. 11.18

From equation (1), we can find stopping potential V_0 for any frequency ν .

Problem 7. Sketch a graph between frequency of incident radiations and stopping potential for a given photosensitive material. What information can be obtained from the value of intercept on the potential axis?

A source of light of frequency greater than the threshold frequency is placed at a distance of 1 m from the cathode of a photo-cell. The stopping potential is found to be V . If the distance of the light source from the cathode is reduced, explain giving reasons, what change will you observe in the

- (i) photoelectric current,
- (ii) stopping potential?

Solution. The graph between frequency of incident radiation and stopping potential for a given photosensitive surface is shown in Fig. 11.18.

From the value of the intercept on the potential axis, we can determine the work function W_0 as follows :

$$\text{As } eV_0 = hv - W_0$$

$$\text{or } V_0 = \left(\frac{h}{e}\right)v - \frac{W_0}{e}$$

$$\therefore \text{Intercept on the potential axis} = -\frac{W_0}{e}$$

Work function,

$$W_0 = e \times \text{Magnitude of intercept on the potential axis.}$$

If the distance of the light source from the cathode is reduced, then

- the photoelectric current increases because the intensity of incident radiation increases, and
- the stopping potential V_0 remains the same because it is independent of the intensity.

Problem 8. When a given photosensitive material is irradiated with light of frequency ν , the maximum speed of the emitted photoelectrons equals v_{\max} . The square of v_{\max} , i.e., v_{\max}^2 , is observed to vary with ν , as per the graph shown in Fig. 11.19. Obtain expressions for (i) Planck's constant, and (ii) the work function of the given photosensitive material, in terms of the parameters l , n and the mass, m , of the electron.

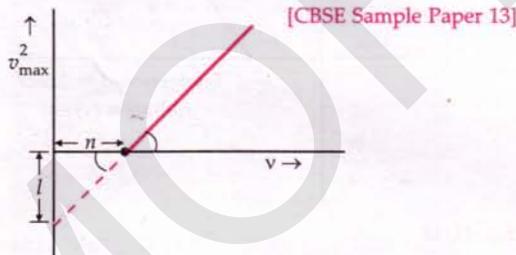


Fig. 11.19

Solution. According to Einstein's photoelectric equation,

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = hv - W_0$$

$$\therefore v_{\max}^2 = \left(\frac{2h}{m}\right)v - \frac{2W_0}{m}$$

Thus the graph of v_{\max}^2 vs. ν is a straight line.

Clearly,

$$\text{Slope of graph, } \frac{2h}{m} = \frac{l}{n}$$

$$\text{Intercept on } v_{\max}^2 \text{ axis, } \frac{2W_0}{m} = l$$

$$\therefore \text{Planck's constant, } h = \frac{lm}{2n}$$

$$\text{Work function, } W_0 = \frac{ml}{2}$$

Problem 9. Sketch the graphs, showing the variation of stopping potential with frequency of incident radiations for two photosensitive materials A and B having threshold frequencies $\nu'_0 > \nu_0$, respectively.

- Which of the two metals, A or B has higher work function? [CBSE OD 14]
- What information do you get from the slope of the graphs?
- What does the value of the intercept of graph 'A' on the potential-axis represent? [CBSE D 06C]

Solution. The graph is shown in Fig. 11.20.

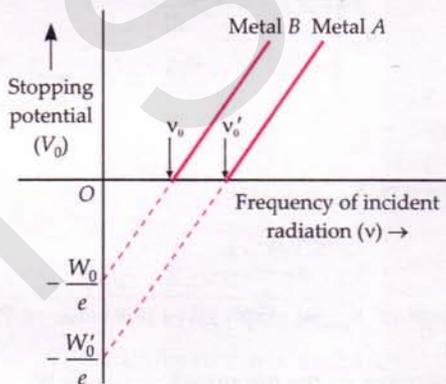


Fig. 11.20

(i) Metal A has a higher work function ($W'_0 = h\nu'_0$) because $\nu'_0 > \nu_0$.

$$(ii) \text{ As } eV_0 = hv - W_0 \quad \text{or} \quad V_0 = \left(\frac{h}{e}\right)v - \frac{W_0}{e}$$

$$\therefore \text{Slope of } V_0 - \nu \text{ graph} = \frac{h}{e}$$

$$(iii) \text{ Intercept of graph A on the potential-axis} \\ = -\frac{W'_0}{e} = -\frac{h\nu'_0}{e}$$

Problem 10. Figure 11.21 shows the variation of stopping potential V_0 with the frequency ν of the incident radiation for two photosensitive metals P and Q :

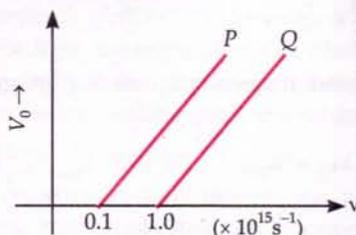


Fig. 11.21

- (i) Explain which metal has smaller threshold wavelength.
- (ii) Explain, giving reason, which metals emits photo electrons having smaller kinetic energy, for the same wavelength of incident radiation.
- (iii) If the distance between the light source and metal P is doubled, how will the stopping potential change? [CBSE OD 08]

Solution. (i) Threshold wavelength, $\lambda_0 = \frac{c}{\nu_0}$

As $\nu_0(Q) > \nu_0(P)$

$\therefore \lambda_0(Q) < \lambda_0(P)$

Thus the metal Q has smaller threshold wavelength.

(ii) According to Einstein's photoelectric equation,

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \text{K.E. of photoelectron}$$

For the same λ of incident radiation, L.H.S. is constant. So metal Q with smaller value of λ_0 will emit photoelectrons of smaller K.E.

(iii) Stopping potential V_0 will remain same because it is independent of intensity and hence of distance between the light source and the metal surface.

Problem 11. Figure 11.22 shows variation of stopping potential (V_0) with the frequency (ν) for two photosensitive materials M_1 and M_2 .

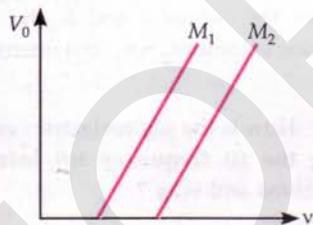


Fig. 11.22

- (i) Why is the slope same for both lines?
- (ii) For which material will the emitted electrons have greater kinetic energy for the incident radiations of the same frequency? Justify your answer. [CBSE F 09 ; OD 15C]

Solution. (i) The slope V_0 - ν graph gives the value of $\frac{h}{e}$, which is same for both materials M_1 and M_2 .

(ii) According to Einstein's photoelectric equation,

$$K_{\max} = h\nu - W_0 = h\nu - h\nu_0$$

The material M_1 has a lower value of threshold frequency ν_0 . So M_1 will emit photoelectrons of greater kinetic energy for the same frequency ν of the incident radiation.

Problem 12. Figure 11.23(a) shows the variation of the stopping potential V_0 with the frequency (ν) of the incident radiations for two different photosensitive materials M_1 and M_2 .

- (i) What are the values of work functions for M_1 and M_2 ?
- (ii) The values of the stopping potential for M_1 and M_2 for a frequency ν_3 ($> \nu_{02}$) of the incident radiations are V_1 and V_2 respectively. Show that the slope of the lines equals $\frac{V_1 - V_2}{\nu_{02} - \nu_{01}}$. [CBSE Sample Paper 08]

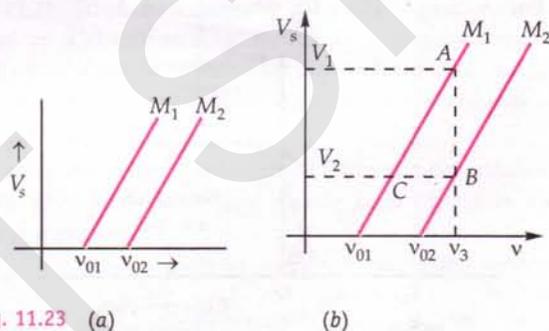


Fig. 11.23 (a)

(b)

Solution. (i) Work function of metal M_1 , $W_1 = h\nu_{01}$
Work function of metal M_2 , $W_2 = h\nu_{02}$

(ii) From Fig. 11.23(b), slope of V - ν lines

$$= \frac{AB}{BC} = \frac{V_1 - V_2}{\nu_{02} - \nu_{01}}$$

Problem 13. Plot a graph showing the variation of stopping potential with the frequency of incident radiation for two different photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$). On what factors does the (i) slope and intercept of the lines depend? [CBSE D 10]

Solution. The graph of V_0 versus ν is shown in Fig. 11.24.

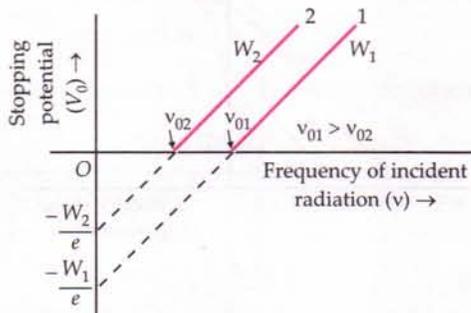


Fig. 11.24

(i) Slope of V_0 - ν graph = $\frac{\Delta V}{\Delta \nu} = \frac{h}{e}$ [$\because e\Delta V = h\Delta \nu$]

Clearly, slope of V_0 - ν graph depends on h and e . It has same value for both materials.

(ii) Intercept on potential axis = $-\frac{W}{e} = -\frac{h\nu_0}{e}$, which depends on the work function of the material.

Problem 14. Draw a plot showing the variation of photoelectric current with collector plate potential for two different frequencies, $\nu_1 > \nu_2$, of incident radiation having the same intensity. In which case will the stopping potential be higher? Justify your answer.

[CBSE OD 11, 14C]

Solution. The plot is shown in Fig. 11.25.

For $\nu_1 > \nu_2$, $V_{01} > V_{02}$

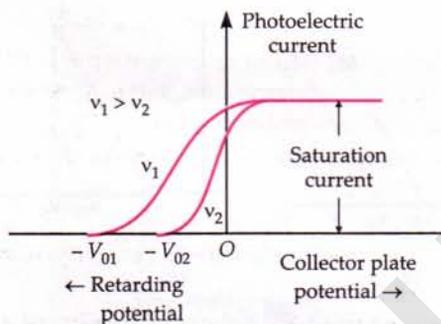


Fig. 11.25

The stopping potential increases linearly with the increase in frequency of incident radiation. Larger the frequency of incident photon, larger is the kinetic energy of the ejected photoelectron and higher is the value of retarding potential required to stop the emission of this electron.

Problem 15. The graph of Fig. 11.26 shows variation of photoelectric current with collector plate potential for different frequencies of incident radiations.

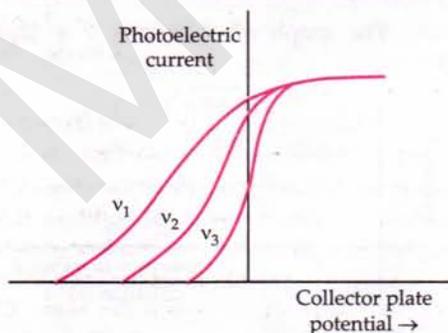


Fig. 11.26

(i) Which physical parameter is kept constant for the three curves?

(ii) Which frequency (ν_1, ν_2 or ν_3) is the highest?

[CBSE F 09]

Solution. (i) Intensity has been kept constant because the saturation current is same for three different frequencies of incident radiation.

(ii) As stopping potential is highest for the radiation of frequency ν_1 , so frequency ν_1 is highest.

Problem 16. Figure 11.27 shows a plot of three curves, a, b, c showing the variation of photocurrent *vs.* collector plate potential for three different intensities I_1, I_2 and I_3 having frequencies ν_1, ν_2 and ν_3 respectively incident on a photosensitive surface.

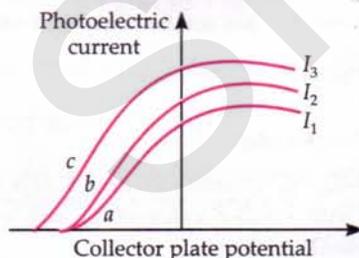


Fig. 11.27

Point out the two curves for which the incident radiations have same frequency but different intensities.

[CBSE D 09]

Solution. For the curves a and b , the stopping potential is same. Hence for curves a and b , the frequency of incident radiation is same ($\nu_1 = \nu_2$) but intensities I_1 and I_2 are different.

Problem 17. How is the photoelectric current affected on increasing the (i) frequency (ii) intensity of the incident radiations and why?

[CBSE OD 06]

Solution. (i) The increase of frequency of incident radiation has no effect on the photoelectric current. This is because the incident photon of increased energy cannot eject more than one electron from the metal surface.

(ii) The photoelectric current increases proportionally with the increase in intensity of incident radiation. Larger the intensity of incident radiation, larger is the number of incident photons and hence larger is the number of electrons ejected from the metallic surface.

Problem 18. Two monochromatic radiations of frequencies ν_1 and ν_2 ($\nu_1 > \nu_2$) and having the same intensity are, in turn, incident on a photosensitive surface to cause photoelectric emission. Explain, giving reason, in which case (i) more number of electrons will be emitted and (ii) maximum kinetic energy of the emitted photoelectrons will be more.

[CBSE D 14C]

Solution. (i) In both cases, the same number of electrons will be emitted because the intensity of the two incident radiations is the same.

(ii) As $K_{\max} \propto \nu$, the maximum K.E. of the photoelectrons will be more in case of radiation with frequency ν_1 .

Problem 19. For a photosensitive surface, threshold wavelength is λ_0 . Does photo-emission occur if the wavelength (λ) of the incident radiation is (i) more than λ_0 , (ii) less than λ_0 ? Justify your answer. [CBSE OD 01C]

Solution. The maximum kinetic energy of the photo-electron is

$$K_{\max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = hc \left(\frac{\lambda_0 - \lambda}{\lambda \lambda_0} \right)$$

- (i) When $\lambda > \lambda_0$, the kinetic energy of the photo-electron is negative. The photo-electric emission does not occur.
- (ii) When $\lambda < \lambda_0$, the kinetic energy of the electron is positive. The photo-electric emission will occur.

Problem 20. Write Einstein's photoelectric equation and mention which important features in photoelectric effect can be explained with the help of this equation.

The maximum kinetic energy of the photoelectrons gets doubled when the wavelength of light incident on the surface changes from λ_1 to λ_2 . Derive the expressions for the threshold wavelength λ_0 and work function for the metal surface. [CBSE D 15]

Solution. According to Einstein's photoelectric equation,

$$\begin{aligned} \text{Energy of incident photon} \\ = \text{Maximum K.E. of photoelectron} \\ + \text{Work function of metal} \end{aligned}$$

$$\text{or } h\nu = K_{\max} + W_0 = \frac{1}{2}mv_{\max}^2 + h\nu_0$$

This equation explains the following important features :

- (i) K_{\max} depends linearly on frequency ν .
- (ii) Existence of threshold frequency for any metal surface.
- (iii) K_{\max} does not depend on the intensity of incident light.

Derivation for λ_0 and W_0 . According to Einstein's photoelectric equation,

$$h\nu = W_0 + K_{\max} = h\nu_0 + K_{\max}$$

$$\text{or } \frac{hc}{\lambda} = \frac{hc}{\lambda_0} + K_{\max}$$

For wavelength λ_1 ,

$$\frac{hc}{\lambda_1} = \frac{hc}{\lambda_0} + K_{\max} \quad \dots(i)$$

For wavelength λ_2 ,

$$\frac{hc}{\lambda_2} = \frac{hc}{\lambda_0} + 2K_{\max} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{2hc}{\lambda_1} - \frac{hc}{\lambda_2} = \frac{hc}{\lambda_0} \quad \text{or} \quad \frac{2}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{\lambda_0}$$

$$\therefore \lambda_0 = \frac{\lambda_1 \lambda_2}{2\lambda_2 - \lambda_1}$$

Work function,

$$W_0 = \frac{hc}{\lambda_0} = \frac{hc(2\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}$$

Problem 21. A beam of monochromatic radiation is incident on a photosensitive surface. Answer the following questions giving reasons :

- (i) Do the emitted photoelectrons have the same kinetic energy?
- (ii) Does the kinetic energy of the emitted electrons depend on the intensity of incident radiation?
- (iii) On what factors does the number of emitted photoelectrons depend? [CBSE F 15]

Solution. (i) No, the different electrons belong to different energy levels in the conduction band. They need different energies to come out of the metal surface. For the same incident radiation, electrons knocked off from different energy levels come out with different energies.

(ii) No, the kinetic energy of a photoelectron depends on the energy of each incident photon and not on the number of photons or intensity of light.

(iii) Number of photoelectrons emitted depends on the intensity of incident light. Larger the intensity of incident radiation, larger is the number of incident photons and hence larger is the number of electrons ejected from the metal surface.

Problem 22. Light of intensity 'I' and frequency ' ν ' is incident on a photosensitive surface and causes photoelectric emission. What will be the effect on anode current when (i) the intensity of light is gradually increased, (ii) the frequency of incident radiation is increased, and (iii) the anode potential is increased? In each case, all other factors remain the same. Explain, giving justification in each case. [CBSE OD 15]

Solution. (i) The anode current increases with the increase in intensity of incident light.

. Larger the intensity of incident radiation, larger is the number of incident photons and hence larger is the number of electrons ejected from the photosensitive surface.

(ii) The increase in the frequency of incident radiation has no effect on the anode current.

. The incident photon of increased energy cannot eject more than one electron from the metal surface. It only increases the maximum K.E. of the ejected electron.

(iii) Anode current first increases with the increase in anode potential and then attains a saturation value.

. All the photoelectrons emitted from the metal do not have the same K.E. The increase in anode potential accelerates more and more electrons towards the anode. A stage is reached when the anode current attains a saturation value. This happens when all the electrons emitted by the metal get collected by the anode.

State how in a photo-cell, the work function of the metal influence the kinetic energy of emitted electrons.

(a) If the intensity of incident radiation is doubled, what changes occur in

- (i) the stopping potential and
- (ii) the photoelectric current ?

(b) If the frequency of the incident radiation is doubled, what changes occur in the

- (i) stopping potential and
- (ii) photoelectric current ?

K.E. of the emitted electrons,

$$K_{\max} = \frac{1}{2} m v_{\max}^2 = h\nu - W_0$$

Higher is the work function (W_0) of the metal, the lesser will be the K.E. of the emitted electrons.

- (a) If the intensity of the radiation is doubled,
 - (i) stopping potential remains unchanged and
 - (ii) the photoelectric current gets doubled.
- (b) If the frequency of the incident radiation is doubled,
 - (i) the stopping potential gets doubled and
 - (ii) the photoelectric current remains unaffected.

If the frequency of the incident radiation on the cathode of a photo-cell is doubled, how will the following change :

- (i) Kinetic energy of the electrons ?
- (ii) Photoelectric current ?
- (iii) Stopping potential ?

Justify your answer.

(i) The K.E. of the photoelectron becomes more than double of its original energy. As the work function of the metal is fixed, so incident photon of higher energy will impart more energy to the photoelectron.

(ii) The increase in frequency of incident radiation has no effect on photoelectric current. This is because of incident photon of increased energy cannot eject more than one electron from the metal surface.

(iii) With the increase in frequency, the K.E. of the photoelectron increases, so stopping potential also increases.

Radiation of frequency 10^{15} Hz is incident on three photo-sensitive surfaces A, B and C. Following observations are recorded :

Surface A : No photo-emission occurs.

Surface B : Photo-emission occurs but the photo-electrons have zero kinetic energy.

Surface C : Photo-emission occurs and photo-electrons have some K.E.

Based on Einstein's photo-electric equation, explain the three observations.

From the observations made (parts A and B) on the basis of Einstein's photoelectric equation, we draw following conclusions :

1. For surface A, the threshold frequency is more than 10^{15} Hz, hence no photo-emission is possible.
2. For surface B, the threshold frequency is equal to the frequency of given radiation. Thus, photo-emission takes place but kinetic energy of photo-electrons is zero.
3. For surface C, the threshold frequency is less than 10^{15} Hz. So photo-emission occurs and photo-electrons have some kinetic energy.

Radiations of frequency 10^{15} Hz are incident on two photo-sensitive surfaces P and Q. Following observations are made :

- (i) **Surface P : Photo-emission occurs but the photo-electrons have zero kinetic energy, and**
- (ii) **Surface Q : Photo-emission occurs and photo-electrons have some kinetic energy.**

Which of these has a higher work function ? If the incident frequency is slightly reduced, what will happen to photo-electron emission in the two cases ?

Solution. By Einstein's photo-electric equation,

$$W_0 = h\nu - K_{\max}$$

For surface P, the photo-electrons have zero kinetic energy while for surface Q, the photo-electrons have some kinetic energy. It follows from the above equation that the surface P has a higher work function W_0 than the surface Q.

If the incident frequency ν is slightly reduced, the energy of the incident photon will become less than the work function W_0 of the surface P. There will be no photo-electric emission from surface P. In case of surface Q, the kinetic energy of photo-electrons will decrease.

A proton and an alpha particle are accelerated through the same potential. Which one of the two has (i) greater value of de-Broglie wavelength associated with it, and (ii) less kinetic energy? Justify your answer.

(i) de Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

For same V ,

$$\lambda \propto \frac{1}{\sqrt{mq}}$$

$$\begin{aligned} \therefore \frac{\lambda_p}{\lambda_\alpha} &= \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} \\ &= \sqrt{\frac{4m_p \cdot 2e}{m_p \cdot e}} = \sqrt{8} = 2\sqrt{2} \end{aligned}$$

Hence $\lambda_p > \lambda_\alpha$, i.e., proton has a greater value of de-Broglie wavelength.

(ii) Kinetic energy $K = qV$

For same V ,

$$K \propto q$$

$$\therefore \frac{K_p}{K_\alpha} = \frac{q_p}{q_\alpha} = \frac{e}{2e} = \frac{1}{2}$$

Hence, $K_p < K_\alpha$, i.e., proton has less kinetic energy.

An electron and a proton are accelerated through the same potential. Which one of the two has (i) greater value of de-Broglie wavelength associated with it and (ii) less momentum? Justify your answer.

(i) de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For same V ,

$$\lambda \propto \frac{1}{\sqrt{mq}}$$

$$\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p \times e}{m_e \times e}} = \sqrt{\frac{m_p}{m_e}}$$

As $m_p > m_e$, so $\lambda_e > \lambda_p$, i.e., electron has a greater de-Broglie wavelength.

(ii) Momentum, $p = \sqrt{2meV}$

$$\text{or } p \propto \sqrt{m}$$

$$\text{As } m_e < m_p, \quad p_e < p_p$$

i.e., electron has less momentum.

An electron, α -particle and a proton have the same kinetic energy. Which of these particles has the shortest de-Broglie wavelength?

The kinetic energy of a particle,

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{p^2}{2m}$$

$$\therefore \text{Linear momentum, } p = \sqrt{2mK}$$

$$\text{de-Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

For the particles possessing same kinetic energy,

$$\lambda \propto \frac{1}{\sqrt{m}}$$

$$\text{As } m_e \ll m_p < m_\alpha$$

$$\therefore \lambda_e \gg \lambda_p > \lambda_\alpha$$

Hence the α -particle has the shortest de-Broglie wavelength.

An α -particle and a proton are accelerated through the same potential difference. Calculate the ratio of linear momenta acquired by the two.

The kinetic energy gained by a particle when accelerated through potential difference V is

$$\frac{1}{2}mv^2 = qV$$

$$\text{or } m^2v^2 = 2mqV$$

\therefore Momentum,

$$p = mv = \sqrt{2mqV}$$

$$\therefore \frac{p_\alpha}{p_p} = \sqrt{\frac{2m_\alpha q_\alpha V}{2m_p q_p V}}$$

$$= \sqrt{\frac{2 \times 4m_p \times 2e \times V}{2 \times m_p \times e \times V}} = 2\sqrt{2} : 1.$$

Mention the significance of Davisson-Germer experiment. An α -particle and a proton are accelerated from rest through the same potential difference V . Find the ratio of de-Broglie wavelengths associated with them.

Davisson-Germer experiment establishes the existence of matter waves of electrons. The kinetic energy gained by a charged particle when accelerated through a potential difference of V volts is given by

$$\frac{1}{2}mv^2 = qV$$

$$\text{or } m^2v^2 = 2mqV$$

$$\text{or } p = mv = \sqrt{2mqV}$$

$$\therefore \text{de Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

For the same potential difference V ,

$$\frac{\lambda(\alpha\text{-particle})}{\lambda(\text{proton})} = \frac{\sqrt{m_p q_p}}{\sqrt{m_\alpha q_\alpha}} = \frac{\sqrt{m_p \times e}}{\sqrt{4m_p \times 2e}} = \frac{1}{2\sqrt{2}}$$

Problem 32. The two lines A and B shown in the graph plot the de-Broglie wavelength (λ) as a function of $1/\sqrt{V}$ (V is the accelerating potential) for two particles having the same charge. Which of the two represents the particle of heavier mass? [CBSE D 04C ; OD 08]

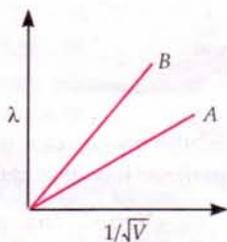


Fig. 11.28

Solution. de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{h}{\sqrt{2mq}} \cdot \frac{1}{\sqrt{V}}$$

$$\therefore \text{Slope of } \lambda \text{ versus } 1/\sqrt{V} \text{ graph} = \frac{h}{\sqrt{2mq}}$$

For the particles of same charge q ,

$$\text{slope} \propto \frac{1}{\sqrt{m}}$$

As the slope of line A is smaller than that of line B, so the line A represents the heavier particle.

Problem 33. A proton and an α -particle have the same de-Broglie wavelength. Determine the ratio of (i) their accelerating potentials (ii) their speeds. [CBSE D 15]

$$\text{Solution. (i) } \lambda = \frac{h}{\sqrt{2mqV}} \Rightarrow V = \frac{h^2}{2mq\lambda^2}$$

$$\begin{aligned} \text{As } m_\alpha &= 4m_p \text{ and } q_\alpha = 2q_p \\ \therefore \frac{V_p}{V_\alpha} &= \frac{m_\alpha q_\alpha}{m_p q_p} = \frac{4m_p \times 2q_p}{m_p q_p} = 8 : 1 \end{aligned}$$

$$\text{(ii) } \lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$$

$$\therefore \frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p} = 4 : 1$$

Problem 34. What reasoning led de-Broglie to put forward the concept of matter waves? The wavelength, λ , of a photon, and the de-Broglie wavelength associated

with a particle of mass ' m ', has the same value, say λ . Show that the energy of photon is $\frac{2\lambda mc}{h}$ times the kinetic energy of the particle. [CBSE Sample Paper 11]

Solution. de-Broglie put forward the bold hypothesis that moving particles of matter should display wave-like properties under suitable conditions. He reasoned that nature was symmetrical and that the two basic physical entities, matter and energy, must have symmetrical character. If radiation shows a dual nature, so should matter.

For a particle of mass m ,

$$K = \frac{p^2}{2m} \text{ and } p = \frac{h}{\lambda}$$

$$\therefore K = \frac{h^2}{2m\lambda^2}$$

$$\text{Also, } E_{\text{photon}} = \frac{hc}{\lambda}$$

$$\therefore \frac{K}{E_{\text{photon}}} = \frac{h}{2mc\lambda}$$

$$\text{or } E_{\text{photon}} = \left(\frac{2\lambda mc}{h} \right) K$$

Problem 35. An electron and a photon have same de-Broglie wavelength (say λ). Which one possesses more kinetic energy? [Himachal 95, 98]

Or

Compare the energy of an electron of de-Broglie wavelength 1 \AA with that of an X-ray photon of the same wavelength. [CBSE Sample Paper 03]

Solution. de-Broglie wavelength of electron, $\lambda = \frac{h}{p}$

$$\therefore \text{Momentum of an electron, } p = \frac{h}{\lambda}$$

$$\text{K.E. of an electron} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

The energy of X-ray photon is totally kinetic.

$$\text{Energy of X-ray photon} = hv = \frac{hc}{\lambda}$$

$$\therefore \frac{\text{Electron energy}}{\text{Energy of X-ray photon}}$$

$$= \frac{h^2}{2m\lambda^2} \times \frac{\lambda}{hc} = \frac{h}{2m\lambda c}$$

$$= \frac{6.6 \times 10^{-34}}{2 \times 9.11 \times 10^{-31} \times 1 \times 10^{-10} \times 3 \times 10^8}$$

$$= \frac{11}{911} < 1$$

Thus the K.E. of a photon is greater than that of an electron of same wavelength.

Problem 36. An electron and a photon each have de-Broglie wavelength of 1.00 nm. (i) Write the ratio of their linear momenta. (ii) Compare the energy of the photon with the kinetic energy of the electron.

[CBSE D 09C]

Solution. (i) As both, electron and photon have same wavelength, so they have same momentum also.

$$\therefore p_e : p_p = 1 : 1$$

(ii) From the solution of Problem 35,

$$\begin{aligned} \frac{\text{Energy of photon}}{\text{K.E. of electron}} &= \frac{2m\lambda c}{h} \\ &= \frac{2 \times 9.11 \times 10^{-31} \times 10^{-9} \times 3 \times 10^8}{6.6 \times 10^{-34}} \\ &= \frac{9110}{11} = 9110 : 11. \end{aligned}$$

Problem 37. Calculate the ratio of the accelerating potentials required to accelerate (i) a proton and (ii) an α -particle to have the same de-Broglie wavelength associated with them.

[Given : Mass of proton = 1.6×10^{-27} kg ; Mass of α -particle = 6.4×10^{-27} kg] [CBSE D 09C]

Solution. If a particle is accelerated through a potential difference V , then

$$qV = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\text{or } p = \sqrt{2mqV}$$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

$$\text{As } \lambda_p = \lambda_\alpha$$

$$\text{or } \frac{h}{\sqrt{2m_p q_p V_p}} = \frac{h}{\sqrt{2m_\alpha q_\alpha V_\alpha}}$$

$$\text{or } \frac{V_p}{V_\alpha} = \frac{m_\alpha q_\alpha}{m_p q_p} = \frac{4m_p \cdot 2e}{m_p \cdot e} = 8 : 1.$$

Problem 38. An electron and a proton have the same de-Broglie wavelength. Which one of these has higher kinetic energy? Which one is moving faster? [ISCE 94]

Solution. de-Broglie wavelength of a particle of mass m and kinetic energy K is

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$$\text{or } \lambda^2 = \frac{h^2}{2mK}$$

$$\therefore K = \frac{h^2}{2m\lambda^2}$$

For particles having same λ , $K \propto \frac{1}{m}$

As the mass of electron is smaller than that of proton, so the electron has a higher kinetic energy and it is moving faster.

Problem 39. An electron and a proton have same wavelength. Which one possesses more energy?

Solution. The total relativistic energy of a particle is

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

As wavelength λ is same for both electron and proton,

\therefore Momentum, $p = \frac{h}{\lambda}$ is same for both particles and hence $p^2 c^2$ is same for both.

But rest mass m_0 of a proton is greater than that of an electron, therefore, the energy of a proton is more than that of an electron of same wavelength.

Problem 40. An electron and photon have same wavelength. Which one of the two has more energy?

Solution. Momentum, $p = \frac{h}{\lambda}$

As both electron and photon have same wavelength, so they have equal momentum.

Relativistic energy of a particle,

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

But for a photon, $m_0 = 0$ and for an electron, $m_0 > 0$.

Hence the electron has more energy than the photon.

Problem 41. The speed of electrons in an electron microscope is 10^8 ms^{-1} . If protons with the same speed are used instead of electrons, what additional advantage such a proton microscope has over an electron microscope?

Solution. de-Broglie wavelength,

$$\lambda = \frac{h}{mv}$$

As the mass of a proton is 1836 times that of an electron, so for the same speed, the wavelength of proton beam will be $\frac{1}{1836}$ times that of an electron beam.

But

$$\text{resolving power of a microscope} \propto \frac{1}{\text{wavelength}}$$

Hence the resolving power of a proton microscope will be 1836 times that of an electron microscope.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. Find the number of photons emitted per second by a 25 W source of monochromatic light of wavelength 6600 Å. What is the photoelectric current assuming 3% efficiency for photoelectric effect? Given $h = 6.6 \times 10^{-34}$ Js.

Solution. Energy of each photon

$$= \frac{hc}{\lambda}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6600 \times 10^{-10}}$$

$$= 3 \times 10^{-19} \text{ J}$$

Total energy emitted per second by 25 W source

$$= 25 \text{ J}$$

Number of photons emitted per second

$$= \frac{25}{3 \times 10^{-19}} = 8.33 \times 10^{19}$$

Photoelectric current

$$= 3\% \text{ of photons emitted per second} \times \text{charge on electron}$$

$$= \frac{3}{100} \times \frac{25}{3 \times 10^{-19}} \times 1.6 \times 10^{-19} = 0.4 \text{ A.}$$

Problem 2. A blue lamp mainly emits light of wavelength 4500 Å. The lamp is rated at 150 W and 8% of the energy is emitted as visible light. How many photons are emitted by the lamp per second?

Solution. $N = \frac{8\% \text{ of } P}{E} = \frac{8P\lambda}{100hc}$

$$= \frac{8 \times 150 \times 4500 \times 10^{-10}}{100 \times 6.63 \times 10^{-34} \times 3 \times 10^8}$$

$$= 2.71 \times 10^{29} \text{ photons/second.}$$

Problem 3. Define the term 'work function' of a metal. The threshold frequency of a metal is f_0 . When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of electrons emitted is v_1 . When the frequency of the incident radiation is increased to $5f_0$, the maximum velocity of electrons emitted is v_2 . Find the ratio of v_1 to v_2 .

[CBSE D 04]

Solution. The minimum amount of energy required to eject an electron from a metal surface (without imparting it any kinetic energy) is called work function of the metal. As f_0 is the threshold frequency, so

$$W = hf_0$$

From Einstein's photoelectric equation, the maximum kinetic energy of emitted electron is given by

$$\frac{1}{2} mv_1^2 = h \times 2f_0 - W = h \times 2f_0 - hf_0 = hf_0$$

and $\frac{1}{2} mv_2^2 = h \times 5f_0 - hf_0 = 4hf_0$

$$\therefore \frac{\frac{1}{2} mv_1^2}{\frac{1}{2} mv_2^2} = \frac{hf_0}{4hf_0}$$

or $\frac{v_1}{v_2} = \frac{1}{2} = 1 : 2.$

Problem 4. If the frequency of incident light on a metal surface is doubled, will the kinetic energy of the photoelectrons be doubled? Give reason.

Solution. Let W_0 be the work function of the metal. Let E_1 and E_2 be the kinetic energies of photoelectrons corresponding to frequencies ν and 2ν of the incident radiation.

Using Einstein's photoelectric equation, we find

$$h\nu = E_1 + W_0$$

and $2h\nu = E_2 + W_0$

On dividing,

$$2 = \frac{E_2 + W_0}{E_1 + W_0}$$

or $2E_1 + 2W_0 = E_2 + W_0$

or $E_2 = 2E_1 + W_0$

Thus kinetic energy of photoelectrons is increased more than double on doubling the frequency of incident radiation.

Problem 5. On the basis of photon theory, obtain Einstein's photo-electric equation. Use this equation to show that there must exist a threshold frequency for each photo-sensitive surface. Radiations of frequencies ν_1 and ν_2 are made to fall in turn, on a photo-sensitive surface. The stopping potentials required for stopping the most energetic emitted photo-electrons in the two cases are V_1 and V_2 respectively. Obtain a formula for determining Planck's constant and the threshold frequency in terms of these parameters.

[CBSE Sample Paper 03]

Solution. For Einstein's photo-electric equation, refer to solution of Problem 3 on page 11.28.

Expression for Planck's constant. Let W_0 be the work function of the metal. Then maximum kinetic energies of the photoelectrons ejected in the two cases will be

$$eV_1 = hv_1 - W_0 \quad \text{and} \quad eV_2 = hv_2 - W_0$$

$$\therefore e(V_2 - V_1) = h(v_2 - v_1)$$

or
$$h = \frac{e(V_2 - V_1)}{v_2 - v_1}$$

If v_0 is the threshold frequency, then

$$eV_1 = hv_1 - W_0 = hv_1 - hv_0$$

or
$$v_0 = v_1 - \frac{eV_1}{h} = v_1 - \frac{V_1(v_2 - v_1)}{V_2 - V_1}$$

$$= \frac{v_1 V_2 - v_1 V_1 - v_2 V_1 + v_1 V_1}{V_2 - V_1}$$

$$= \frac{v_1 V_2 - v_2 V_1}{V_2 - V_1}$$

Problem 6. Obtain the relationship between stopping potential and frequency of incident radiations for photo-emission.

X-rays of wavelength 0.82 \AA fall on a metallic surface. Calculate the de-Broglie wavelength of the emitted photoelectrons. Neglect the work function of the surface.

[CBSE Sample Paper 03]

Solution. According to Einstein's theory of photo-electric effect,

Energy of incident photon
= Maximum K.E. of emitted photo-electron
+ Work function of metal

$$hv = \frac{1}{2} mv_{\max}^2 + W_0$$

If V_0 is the stopping potential, then

$$\frac{1}{2} mv_{\max}^2 = eV_0$$

$$\therefore hv = eV_0 + W_0$$

or
$$eV_0 = hv - W_0$$

This is the required relation between stopping potential V_0 and the frequency ν of the incident radiation.

Numerical. Here $\lambda = 0.82 \text{ \AA} = 0.82 \times 10^{-10} \text{ m}$, $W_0 = 0$

From Einstein's photo-electric equation,

K.E. of photo-electron,

$$\frac{1}{2} mv^2 = hv - W_0 = hv - 0$$

or
$$mv^2 = 2hv = \frac{2hc}{\lambda}$$

or
$$mv = \sqrt{\frac{2hc}{\lambda}}$$

\therefore de-Broglie wavelength associated with the electron is

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2hc/\lambda}} = \sqrt{\frac{h\lambda}{2c}}$$

$$= \sqrt{\frac{6.6 \times 10^{-34} \times 0.82 \times 10^{-10}}{2 \times 3 \times 10^8 \times 9.1 \times 10^{-31}}}$$

$$= 0.0995 \times 10^{-10} \text{ m}$$

$$= 0.0995 \text{ \AA}$$

Problem 7. X-rays fall on a photosensitive surface to cause photoelectric emission. Assuming that the work function of the surface can be neglected, find the relation between the de-Broglie wavelength (λ) of the electrons emitted to the energy (E_v) of the incident photons. Draw the nature of the graph for λ as a function of E_v . [CBSE D 14C]

Solution. Energy of a photon,

$$E_v = W_0 + K_{\max}$$

As $W_0 = 0$, so $E_v = K_{\max} = \frac{p^2}{2m}$

$$\therefore p = \sqrt{2mE_v}$$

Wavelength of the emitted electrons,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_v}} \quad \text{i.e.,} \quad \lambda \propto \frac{1}{\sqrt{E_v}}$$

Hence, the graph of λ vs. E_v is a parabola as shown in the figure.

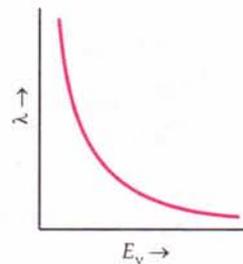


Fig. 11.29

Problem 8. When light of wavelength 400 nm is incident on the cathode of a photocell, the stopping potential recorded is 6 V . If the wavelength of the incident light is increased to 600 nm , calculate the new stopping potential.

[CBSE OD 2000]

Solution. As $K_{\max} = h\nu - W_0$

$$\text{or } eV_0 = \frac{hc}{\lambda} - W_0$$

$$\text{or } V_0 = \frac{hc}{e\lambda} - \frac{W_0}{e}$$

$$\begin{aligned} \therefore \Delta V_0 &= (V_0)_2 - (V_0)_1 \\ &= \left[\frac{hc}{e\lambda_2} - \frac{W_0}{e} \right] - \left[\frac{hc}{e\lambda_1} - \frac{W_0}{e} \right] = \frac{hc}{e} \left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right] \\ &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}} \left[\frac{1}{4 \times 10^{-7}} - \frac{1}{6 \times 10^{-7}} \right] \\ &= 1.03 \end{aligned}$$

$$(V_0)_2 = (V_0)_1 - 1.03 = 6 - 1.03 = 4.97 \text{ V}$$

Problem 9. Light is incident on the cathode of a photocell and the stopping voltages are measured for light of two different wavelengths. From the data given below, determine the work function of the metal of the cathode in eV and the value of universal constant hc/e .

Wavelength (Å)	Stopping voltage (volt)
4000	1.3
4500	0.9

[Roorkee 84]

Solution. From problem 8, we have

$$\Delta V_0 = \frac{hc}{e} \left[\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right]$$

$$\begin{aligned} \therefore \frac{hc}{e} &= \frac{\Delta V_0 \cdot \lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \\ &= \frac{(1.3 - 0.9) \times 4000 \times 10^{-10} \times 4500 \times 10^{-10}}{500 \times 10^{-10}} \\ &= 1.44 \times 10^{-6} \text{ Vm.} \end{aligned}$$

$$\text{Also, } V_0 = \frac{hc}{e\lambda} - \frac{W_0}{e}$$

$$\therefore \frac{W_0}{e} = \frac{hc}{e\lambda} - V_0$$

$$= \frac{1.44 \times 10^{-6}}{4000 \times 10^{-10}} - 1.3 = 2.3 \text{ V}$$

$$\therefore W_0 = 2.3 \text{ eV.}$$

Problem 10. A particle of a mass M at rest decays into two particles of masses m_1 and m_2 having non-zero velocities. What is the ratio of the de Broglie wavelengths of the two particles ?

[IIT 99]

Solution. By conservation of linear momentum,

$$m_1 v_1 + m_2 v_2 = M \times 0$$

$$\text{or } m_1 v_1 = -m_2 v_2$$

$$\therefore |m_1 v_1| = |m_2 v_2|$$

$$\text{or } |p_1| = |p_2|$$

$$\frac{\lambda_1}{\lambda_2} = \frac{|p_2|}{|p_1|} = 1.$$

Problem 11. Red light, however bright it is, cannot produce the emission of electrons from a clean zinc surface. But even weak ultraviolet radiation can do so. Why ?

X-rays of wavelength ' λ ' fall on photo-sensitive surface, emitting electrons. Assuming that the work function of the surface can be neglected, prove that the de-Broglie wavelength of electrons emitted will be $\sqrt{\frac{h\lambda}{2mc}}$.

[CBSE OD 04]

Solution. The frequency of ultraviolet radiations is more while that of red light is less than the threshold frequency for a zinc surface, so ultraviolet radiations can cause emission of electrons and red light cannot.

As the work function of the metal can be neglected, so

K.E. of emitted electron = Energy of X-ray photon

$$\text{or } \frac{1}{2} mv^2 = h\nu$$

$$\text{or } \frac{p^2}{2m} = \frac{hc}{\lambda}$$

$$\text{or } p = \sqrt{\frac{2mhc}{\lambda}}$$

de-Broglie wavelength of emitted electrons,

$$\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{\frac{2mhc}{\lambda}}}$$

$$\text{or } \lambda_1 = \sqrt{\frac{h\lambda}{2mc}}$$

Problem 12. An electromagnetic wave of wavelength λ is incident on a photosensitive surface of negligible work function. If the photoelectrons emitted from this surface have the de-Broglie wavelength λ_1 , prove that $\lambda = \left(\frac{2mc}{h}\right) \lambda_1^2$.

[CBSE D 08]

Solution. As proved in the above problem,

$$\lambda_1 = \sqrt{\frac{h\lambda}{2mc}}$$

or
$$\lambda_1^2 = \frac{h\lambda}{2mc}$$

$$\therefore \lambda = \left(\frac{2mc}{h}\right) \lambda_1^2.$$

Problem 13. Calculate the de-Broglie wavelength of (i) an electron (in the hydrogen atom) moving with a speed of $1/100$ of the speed of light in vacuum and (ii) a ball of radius 5 mm and mass 3×10^{-2} kg moving with a speed of 100 ms^{-1} . Hence show that the wave nature of matter is important at the atomic level but is not really relevant at the macroscopic level.

[CBSE Sample Paper 08]

Solution.

$$(i) \lambda_e = \frac{6.6 \times 10^{-34}}{9 \times 10^{-31} \times \frac{1}{100} \times 3 \times 10^8} \\ = 2.44 \times 10^{-10} \text{ m.}$$

$$(ii) \lambda_b = \frac{6.6 \times 10^{-34}}{3 \times 10^{-2} \times 100} \\ = 2.2 \times 10^{-34} \text{ m}$$

As the de-Broglie wavelength of the electron has a significant value while that of the ball is negligibly small, it shows that the wave nature of matter is important at the atomic level but it is not really relevant at the macroscopic level.

GUIDELINES TO NCERT EXERCISES

11.1. Find the : (a) maximum frequency, and (b) minimum wavelength of X-rays produced by 30 kV electrons.

Ans. (a) Maximum energy of X-ray photon
= Maximum energy of an accelerated electron

$$\text{or } h\nu_{\max} = eV$$

$$\begin{aligned} \therefore \nu_{\max} &= \frac{eV}{h} \\ &= \frac{1.6 \times 10^{-19} \times 30 \times 10^3}{6.63 \times 10^{-34}} \\ &= 7.24 \times 10^{18} \text{ Hz} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \lambda_{\min} &= \frac{c}{\nu_{\max}} = \frac{3 \times 10^8}{7.24 \times 10^{18}} \\ &= 0.0414 \times 10^{-9} \text{ m} \\ &= 0.0414 \text{ nm}. \end{aligned}$$

11.2. The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the :

- maximum kinetic energy of the emitted electrons,
- stopping potential, and
- maximum speed of the emitted photoelectrons ?

Ans. Here $W_0 = 2.14 \text{ eV}$, $\nu = 6 \times 10^{14} \text{ Hz}$

$$\begin{aligned} \text{(a)} \quad K_{\max} &= h\nu - W_0 \\ &= 6.63 \times 10^{-34} \times 6 \times 10^{14} \text{ J} - 2.14 \text{ eV} \\ &= \frac{6.63 \times 6 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} - 2.14 \text{ eV} \\ &= 2.48 - 2.14 = 0.34 \text{ eV}. \end{aligned}$$

$$\text{(b) As } eV_0 = K_{\max} = 0.34 \text{ eV}$$

\therefore Stopping potential,

$$V_0 = 0.34 \text{ V}.$$

$$\begin{aligned} \text{(c)} \quad K_{\max} &= \frac{1}{2} m v_{\max}^2 = 0.34 \text{ eV} \\ &= 0.34 \times 1.6 \times 10^{-19} \text{ J} \end{aligned}$$

or

$$v_{\max}^2 = \frac{2 \times 0.34 \times 1.6 \times 10^{-19}}{m}$$

$$= \frac{2 \times 0.34 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$$

$$= 119560.4 \times 10^6$$

or

$$v_{\max} = 345.8 \times 10^3 \text{ ms}^{-1}$$

$$= 345.8 \text{ kms}^{-1}.$$

11.3. The photoelectric cut-off voltage in a certain experiment is 15 V. What is the maximum kinetic energy of photoelectrons emitted ?

Ans. Here $V_0 = 1.5 \text{ V}$

$$\begin{aligned} K_{\max} &= eV_0 = 1.5 \text{ eV} \\ &= 1.5 \times 1.6 \times 10^{-19} \text{ J} \\ &= 2.4 \times 10^{-19} \text{ J}. \end{aligned}$$

11.4. Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

- Find the energy and momentum of each photon in the light beam.

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Ans. Here $\lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}$,

$$P = 9.42 \text{ mW} = 9.42 \times 10^{-3} \text{ W}$$

(a) Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}}$$

$$= 3.14 \times 10^{-19} \text{ J}$$

Momentum of each photon,

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{632.8 \times 10^{-9}}$$

$$= 1.05 \times 10^{-27} \text{ kg ms}^{-1}$$

(b) Number of photons arriving per second at the target,

$$N = \frac{P}{E} = \frac{9.42 \times 10^{-3}}{3.14 \times 10^{-19}}$$

$$= 3 \times 10^{16} \text{ photons per second.}$$

(c) Momentum of a hydrogen atom

= Momentum of a photon

$$\text{or } mv = p$$

\therefore Velocity,

$$v = \frac{p}{m} = \frac{1.05 \times 10^{-27} \text{ kg ms}^{-1}}{1.67 \times 10^{-27} \text{ kg}} = 0.63 \text{ ms}^{-1}$$

11.5. The energy flux of sunlight reaching the surface of the earth is $1.388 \times 10^3 \text{ Wm}^{-2}$. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Ans. Energy of each photon,

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{550 \times 10^{-9}}$$

$$= 3.62 \times 10^{-19} \text{ J}$$

Number of photons incident on earth's surface per second per square metre

$$= \frac{\text{Total energy per square metre per second}}{\text{Energy of each photon}}$$

$$= \frac{1.388 \times 10^3}{3.62 \times 10^{-19}} = 3.8 \times 10^{21}$$

11.6. In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be $4.12 \times 10^{-15} \text{ Vs}$. Calculate the value of Planck's constant.

Ans. Here $\frac{\Delta V}{\Delta \nu} = 4.12 \times 10^{-15} \text{ Vs}$, $e = 1.6 \times 10^{-19} \text{ C}$

Planck's constant,

$$h = \frac{\Delta V}{\Delta \nu} \cdot e = 4.12 \times 10^{-15} \times 1.6 \times 10^{-19}$$

$$= 6.592 \times 10^{-34} \text{ Js.}$$

11.7. A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

Ans. Here $P = 100 \text{ W}$, $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$

(a) Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{589 \times 10^{-9}} \text{ J}$$

$$= 3.38 \times 10^{-19} \text{ J}$$

$$= \frac{3.38 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 2.11 \text{ eV.}$$

(b) Rate at which photons are delivered to the sphere,

$$N = \frac{P}{E} = \frac{100 \text{ Js}^{-1}}{3.38 \times 10^{-19} \text{ J}}$$

$$= 3.0 \times 10^{20} \text{ photons/ second.}$$

11.8. The threshold frequency for a certain metal is $3.3 \times 10^{14} \text{ Hz}$. If light of frequency $8.2 \times 10^{14} \text{ Hz}$ is incident on the metal, predict the cut of voltage for photoelectric emission. Given $h = 6.63 \times 10^{-34} \text{ Js}$ and $e = 1.6 \times 10^{-19} \text{ C}$.

Ans. Here $\nu_0 = 3.3 \times 10^{14} \text{ Hz}$, $\nu = 8.2 \times 10^{14} \text{ Hz}$, $V_0 = ?$

Maximum K.E. of a photoelectron is

$$eV_0 = h\nu - h\nu_0$$

$$\therefore V_0 = \frac{h(\nu - \nu_0)}{e}$$

$$= \frac{6.63 \times 10^{-34} \times (8.2 - 3.3) \times 10^{14}}{1.6 \times 10^{-19}}$$

$$= 2.03 \text{ V.}$$

11.9. The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Ans. Here $W_0 = 4.2 \text{ eV}$, $\lambda = 330 \text{ nm} = 330 \times 10^{-9} \text{ m}$

Energy of incident photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} \text{ J}$$

$$= \frac{6.63 \times 3 \times 10^{-17}}{330 \times 1.6 \times 10^{-19}} \text{ eV} = 3.767 \text{ eV}$$

As the energy of incident photon is less than the work function of the metal, there will be no photoelectric emission.

Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^5 m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Here $\nu = 7.21 \times 10^{14}$ Hz, $v_{\max} = 6.0 \times 10^5$ ms⁻¹

From Einstein's photoelectric equation,

$$K_{\max} = \frac{1}{2} m v_{\max}^2$$

$$= h\nu - W_0 = h(\nu - \nu_0)$$

$$\therefore \nu - \nu_0 = \frac{\frac{1}{2} m v_{\max}^2}{h}$$

$$= \frac{1 \times 9.1 \times 10^{-31} \times (6.0 \times 10^5)^2}{2 \times 6.63 \times 10^{-34}}$$

$$= 2.47 \times 10^{14} \text{ Hz}$$

or $\nu_0 = \nu - 2.47 \times 10^{14}$

$$= 7.21 \times 10^{14} - 2.47 \times 10^{14}$$

$$= 4.74 \times 10^{14} \text{ Hz.}$$

Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the cathode, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the cathode is made.

Here $\lambda = 488 \text{ nm} = 488 \times 10^{-9} \text{ m}$, $V_0 = 0.38 \text{ V}$

From Einstein's photoelectric equation,

$$K_{\max} = \frac{1}{2} m v_{\max}^2 = h\nu - W_0$$

or $eV_0 = \frac{hc}{\lambda} - W_0$

$$\therefore W_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{488 \times 10^{-9}} - 1.6 \times 10^{-19} \times 0.38$$

$$= 4.076 \times 10^{-19} - 0.608 \times 10^{-19}$$

$$= 3.468 \times 10^{-19} \text{ J}$$

or $W_0 = 3.47 \times 10^{-19} \text{ J} = 2.17 \text{ eV.}$

Calculate the : (a) momentum, and (b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

(a) Kinetic energy of an electron,

$$K = 56 \text{ eV} = 56 \times 1.6 \times 10^{-19} \text{ J}$$

Momentum of an electron,

$$p = \sqrt{2mK}$$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 56 \times 1.6 \times 10^{-19}}$$

$$= \sqrt{1630.72 \times 10^{-50}}$$

$$= 4.04 \times 10^{-24} \text{ kg ms}^{-1}.$$

(b) de Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{4.04 \times 10^{-24}}$$

$$= 0.164 \times 10^{-9} \text{ m}$$

$$= 0.164 \text{ nm.}$$

11.13. What is the (a) momentum, (b) speed, and (c) de Broglie wavelength of an electron with kinetic energy of 120 eV?

Ans. Kinetic energy,

$$K = 120 \text{ eV} = 120 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 1.92 \times 10^{-17} \text{ J}$$

(a) Momentum of an electron,

$$p = \sqrt{2mK} = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.92 \times 10^{-17}}$$

$$= 5.91 \times 10^{-24} \text{ kg ms}^{-1}.$$

(b) Speed of an electron,

$$v = \frac{p}{m} = \frac{5.91 \times 10^{-24}}{9.1 \times 10^{-31}} = 6.5 \times 10^6 \text{ ms}^{-1}.$$

(c) de Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{5.91 \times 10^{-24}}$$

$$= 0.112 \times 10^{-9} \text{ m} = 0.112 \text{ nm.}$$

11.14. The wavelength of light from the spectral emission line of sodium is 589 nm.

Find the kinetic energy at which :

(a) an electron, and

(b) a neutron, would have the same de Broglie wavelength.

Ans. Here $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$

But $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

or $\lambda^2 = \frac{h^2}{2mK}$

$$\therefore K = \frac{h^2}{2m\lambda^2}$$

(a) Kinetic energy of an electron,

$$K = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (589 \times 10^{-9})^2}$$

$$= 6.95 \times 10^{-25} \text{ J} = 4.34 \mu \text{ eV.}$$

(b) Kinetic energy of a neutron,

$$K = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (589 \times 10^{-9})^2}$$

$$= 3.78 \times 10^{-28} \text{ J} = 0.236 \text{ neV.}$$

11.15. What is the de Broglie wavelength of

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,

- (b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and
 (c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s ?

Ans. de Broglie wavelength, $\lambda = \frac{h}{mv}$

$$(a) \lambda_{\text{bullet}} = \frac{6.63 \times 10^{-34}}{0.040 \times 1.0 \times 10^3} = 1.7 \times 10^{-35} \text{ m.}$$

$$(b) \lambda_{\text{ball}} = \frac{6.63 \times 10^{-34}}{0.060 \times 1.0} = 1.1 \times 10^{-32} \text{ m.}$$

$$(c) \lambda_{\text{particle}} = \frac{6.63 \times 10^{-34}}{1.0 \times 10^{-9} \times 2.2} = 3.0 \times 10^{-25} \text{ m.}$$

11.16. An electron and a photon each have a wavelength of 1.00 nm. Find

- (a) their momenta,
 (b) the energy of the photon, and
 (c) the kinetic energy of electron.

(Take $h = 6.63 \times 10^{-34}$ Js)

[CBSE D 11]

Ans. Here $\lambda = 1.00 \text{ nm} = 1.00 \times 10^{-9} \text{ m}$

(a) Both electron and photon have same wavelength, so they have same momentum also.

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{1.00 \times 10^{-9}} = 6.63 \times 10^{-25} \text{ kg ms}^{-1}.$$

(b) Energy of a photon,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.00 \times 10^{-9}} = 19.89 \times 10^{-17} \text{ J} = \frac{19.89 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV} = 1.24 \times 10^3 \text{ eV} = 1.24 \text{ keV.}$$

(c) Kinetic energy of electron,

$$K = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} = 2.42 \times 10^{-19} \text{ J} = \frac{2.42 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.51 \text{ eV.}$$

11.17. (a) For what kinetic energy of a neutron will the associated de Broglie wavelength be $1.40 \times 10^{-10} \text{ m}$?

[CBSE OD 08]

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $3/2 kT$ at 300 K.

Ans. Given $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$

(a) de Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mK}}$

\therefore Kinetic energy,

$$K = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.677 \times 10^{-27} \times (1.40 \times 10^{-10})^2} = 6.686 \times 10^{-21} \text{ J} = \frac{6.686 \times 10^{-21}}{1.6 \times 10^{-19}} = 4.174 \times 10^{-2} \text{ eV.}$$

(b) $K = \frac{3}{2} kT$

$$\therefore \lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{3mkT}} = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.677 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \text{ m} = \frac{6.63 \times 10^{-10}}{\sqrt{20.8}} = \frac{6.63 \times 10^{-10}}{4.56} \text{ m} = 1.45 \times 10^{-10} \text{ m} = 0.145 \text{ nm.}$$

11.18. Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Ans. For a photon,

de Broglie wavelength, $\lambda = \frac{h}{p}$.

For an electromagnetic radiation of frequency ν and wavelength λ' ($= c/\nu$),

Momentum,

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

$$\text{or } p = \frac{h}{c} \cdot \frac{c}{\lambda'} = \frac{h}{\lambda'}$$

$$\text{Then, } \lambda' = \frac{h}{p} = \lambda$$

Thus the wavelength λ' of the electromagnetic radiation is the same as the de Broglie wavelength λ of the photon.

11.19. What is the de Broglie wavelength of a nitrogen molecule in air at 300 K ? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u , $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$).

Ans. Mass of N_2 molecule,

$$m = 2 \times 14.0076 \times 1.66 \times 10^{-27} \text{ kg} = 46.5 \times 10^{-27} \text{ kg}$$

$$T = 300 \text{ K}$$

Average kinetic energy,

$$\frac{1}{2} mc^2 = \frac{3}{2} kT$$

or
$$c = \sqrt{\frac{3kT}{m}}$$

$$\begin{aligned} \therefore \lambda &= \frac{h}{mc} = \frac{h}{\sqrt{3mkT}} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 46.5 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \text{ m} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{577.59 \times 10^{-24}}} \\ &= \frac{6.63 \times 10^{-10}}{24.03} \text{ m} = 0.0276 \times 10^{-9} \text{ m} \approx \mathbf{0.028 \text{ nm}}. \end{aligned}$$

11.20. (a) Estimate the speed with which electrons emitted from a heated cathode of an evacuated tube impinge on the anode maintained at a potential difference of 500 V with respect to the cathode. Ignore the small initial speeds of the electrons. The 'specific charge' of the electron i.e., its e/m is given to be $1.76 \times 10^{11} \text{ C kg}^{-1}$.

(b) Use the same formula you employ in (a) to obtain electron speed for an anode potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?

Ans. (a) Here $V = 500 \text{ V}$, $e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}$, $v = ?$

$$E_k = \frac{1}{2} mv^2 = eV$$

$$\begin{aligned} \therefore v &= \sqrt{\frac{2eV}{m}} \\ &= \sqrt{2 \times 1.7 \times 10^{11} \times 500} = \mathbf{1.33 \times 10^7 \text{ ms}^{-1}}. \end{aligned}$$

(b) Here $V = 10 \text{ MV} = 10^7 \text{ V}$

$$\therefore v = \sqrt{2 \times 1.7 \times 10^{11} \times 10^7} = \mathbf{1.88 \times 10^9 \text{ ms}^{-1}}.$$

This speed is not possible because no particle can have a speed greater than the speed of light ($c = 3 \times 10^8 \text{ ms}^{-1}$). The above formula for K.E. is valid only when $v \ll c$. At speeds comparable to the speed of light, we need to use the relativistic formula,

$$E_k = mc^2 - m_0c^2 = (m - m_0) c^2$$

or
$$eV = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0c^2$$

or
$$\frac{eV}{m_0c^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1$$

or
$$\frac{eV}{m_0c^2} + 1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substituting the various values in L.H.S., we get

$$\frac{1.6 \times 10^{-19} \times 10^7}{9.1 \times 10^{-31} \times (3 \times 10^8)^2} + 1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or
$$19.536 + 1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or
$$1 - \frac{v^2}{c^2} = \frac{1}{(20.536)^2} = 0.00237$$

or
$$\frac{v^2}{c^2} = 1 - 0.00237 = 0.99763$$

or
$$v = \sqrt{0.99763} c = \mathbf{0.999 c}.$$

11.21. (a) A monoenergetic electron beam with electron speed of $5.20 \times 10^6 \text{ ms}^{-1}$ is subject to a magnetic field of $1.30 \times 10^{-4} \text{ T}$ normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals $1.76 \times 10^{11} \text{ C kg}^{-1}$.

(b) Is the formula you employ in (a) valid for calculating radius of the path of a 20 MeV electron beam? If not, in what way is it modified?

Ans. (a) Here $v = 5.20 \times 10^6 \text{ ms}^{-1}$, $B = 1.30 \times 10^{-4} \text{ T}$

$$\frac{e}{m} = 1.76 \times 10^{11} \text{ C kg}^{-1}, \theta = 90^\circ$$

The normal magnetic field provides necessary centripetal force to the electron beam so that it can trace a circular path. Thus

Force on an electron = Centripetal force
due to magnetic field on an electron

or
$$evB \sin 90^\circ = \frac{mv^2}{R}$$

or
$$\begin{aligned} R &= \frac{mv}{eB} = \frac{v}{(e/m)B} \\ &= \frac{5.20 \times 10^6}{1.76 \times 10^{11} \times 1.30 \times 10^{-4}} \text{ m} \\ &= \frac{520}{176 \times 13} \text{ m} = 0.227 \text{ m} \\ &= \mathbf{22.7 \text{ cm}}. \end{aligned}$$

(b) No, the formula: $R = \frac{mv}{eB}$ is not valid for calculating the radius of the path of the electron beam because such high energy electrons have velocities comparable to the speed of light. For this, we should use the relativistic formula,

$$R = \frac{mv}{eB} = \frac{m_0v}{eB \sqrt{1 - \frac{v^2}{c^2}}}$$

11.22. An electron gun with its anode at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure (10^{-2} mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. Determine e/m from the data.

Ans. Here $V = 100$ V, $r = 12$ cm = 12×10^{-2} m, $B = 2.83 \times 10^{-4}$ T

The gain in kinetic energy of an electron when accelerated through V volts is

$$\frac{1}{2}mv^2 = eV \quad \text{or} \quad v^2 = \frac{2eV}{m}$$

As the magnetic field provides centripetal force to the electron, therefore,

$$evB = \frac{mv^2}{r} \quad \text{or} \quad v = \frac{eBr}{m}$$

$$v^2 = \frac{e^2 B^2 r^2}{m^2}$$

$$\frac{2eV}{m} = \frac{e^2 B^2 r^2}{m^2}$$

$$\text{or} \quad \frac{e}{m} = \frac{2V}{B^2 r^2}$$

$$= \frac{2 \times 100}{(2.83 \times 10^{-4})^2 \times (12 \times 10^{-2})^2}$$

$$= 1.73 \times 10^{11} \text{ C kg}^{-1}.$$

11.23. (i) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 \AA . What is the maximum energy of a photon in the radiation? [Punjab 95]

(ii) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Ans. (i) Here $\lambda_{\min} = 0.45 \text{ \AA} = 0.45 \times 10^{-10}$ m

The maximum energy of an X-ray photon is

$$E_{\max} = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.45 \times 10^{-10}} \text{ J}$$

$$= \frac{6.63 \times 3 \times 10^{-16}}{0.45 \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 27.6 \times 10^3 \text{ eV} = 27.6 \text{ keV}.$$

(b) To get photons of energy 27.6 keV, electrons of atleast 27.6 keV must strike the target of the X-ray tube. Hence the accelerating potential should be greater than 27.6 eV or it should be of the order of 30 keV.

11.24. In an accelerator experiment on high energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron positron pair of total energy 10.2 BeV into two γ -rays of equal energy. What is the wavelength associated with each γ -ray? ($1 \text{ BeV} = 10^9 \text{ eV}$).

Ans. Energy of two γ -rays

= Energy of electron-positron pair

$$= 10.2 \text{ BeV} = 10.2 \times 10^9 \text{ eV}$$

\therefore Energy of each γ -ray photon is

$$E = 5.1 \times 10^9 \text{ eV}$$

$$= 5.1 \times 10^9 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 5.1 \times 1.6 \times 10^{-10} \text{ J}$$

$$\text{But} \quad E = h\nu = \frac{hc}{\lambda}$$

Hence wavelength associated with γ -ray is

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5.1 \times 1.6 \times 10^{-10}} \text{ m}$$

$$= 2.436 \times 10^{-16} \text{ m}.$$

11.25. (a) Find the number of photons emitted per second by a MW transmitter of 10 kW power emitting radiowaves of wavelength 500 m.

(b) Find the number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive ($\approx 10^{-10} \text{ Wm}^{-2}$). Take the area of the pupil to be about 0.4 cm^2 , and the average frequency of white light to be about $6 \times 10^{14} \text{ Hz}$.

Ans. (a) Power of transmitter = 10 kW = 10^4 W

Total energy emitted per second

$$= \text{Power} \times \text{time} = 10^4 \text{ W} \times 1 \text{ s} = 10^4 \text{ J}$$

Energy of one photon,

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{500} \text{ J}$$

[Given $\lambda = 500$ m]

Number of photons emitted per second is

$$N = \frac{\text{Total energy emitted per second}}{\text{Energy of one photon}}$$

$$= \frac{10^4 \times 500}{6.63 \times 10^{-34} \times 3 \times 10^8} = 2.51 \times 10^{31}.$$

(b) Minimum intensity, $I = 10^{-10} \text{ Wm}^{-2}$

Area of pupil,

$$A = 0.4 \text{ cm}^2 = 0.4 \times 10^{-4} \text{ m}^2$$

Average frequency, $\nu = 6 \times 10^{14} \text{ Hz}$

Energy of one photon,

$$E = h\nu$$

$$= 6.63 \times 10^{-34} \times 6 \times 10^{14} \text{ J}$$

$$= 4 \times 10^{-19} \text{ J}$$

Let n = number of photons crossing per square metre area per second

$$\text{Now Intensity} = \frac{\text{Energy incident per square metre area per second}}{\text{Total Energy of } n \text{ photons}}$$

$$= n \times \text{energy of one photon}$$

$$\begin{aligned} \therefore n &= \frac{\text{Intensity}}{\text{Energy of one photon}} \\ &= \frac{10^{-10} \text{ Wm}^{-2}}{4 \times 10^{-19} \text{ J}} = 2.5 \times 10^8 \text{ m}^{-2} \text{ s}^{-1} \end{aligned}$$

\therefore The number of photons entering the pupil of our eye per second

$$\begin{aligned} &= n \times \text{Area of the pupil} \\ &= 2.5 \times 10^8 \times 0.4 \times 10^{-4} \text{ s}^{-1} = 10^4 \text{ s}^{-1}. \end{aligned}$$

11.26. Ultraviolet light of wavelength 2271 \AA from a 100 W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is 1.3 V , estimate the work function of the metal. How would the photocell respond to a high intensity ($= 10^5 \text{ Wm}^{-2}$) red light of wavelength 6328 \AA produced by a He-Ne-laser? [CBSE D 05, 11C ; F 13]

Ans. Here $\lambda = 2271 \text{ \AA} = 2271 \times 10^{-10} \text{ m}$, $V_0 = 1.3 \text{ V}$
 $h = 6.63 \times 10^{-34} \text{ Js}$, $e = 1.6 \times 10^{-19} \text{ C}$, $W_0 = ?$

Einstein's photoelectric equation is

Maximum K.E. of emitted photon

$$= eV_0 = hv - W_0$$

$$\therefore W_0 = hv - eV_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2271 \times 10^{-10}} - 1.6 \times 10^{-19} \times 1.3$$

$$= [8.72 \times 10^{-19} - 2.08 \times 10^{-19}] \text{ J}$$

$$= \frac{6.64 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 4.2 \text{ eV} \quad [\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

Threshold frequency is

$$\nu_0 = \frac{W_0}{h} = \frac{6.64 \times 10^{-19}}{6.63 \times 10^{-34}} \text{ Hz}$$

$$= 1.0 \times 10^{15} \text{ Hz}$$

For red light, $\lambda = 6328 \text{ \AA} = 6328 \times 10^{-10} \text{ m}$

\therefore Corresponding frequency for red light will be

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{6328 \times 10^{-10}}$$

$$= 4.74 \times 10^{14} \text{ Hz}$$

As $\nu < \nu_0$, so there will be no photoelectric emission with this red light, howsoever high its intensity may be.

11.27. Monochromatic radiation of wavelength 640.2 nm ($1 \text{ nm} = 10^{-9} \text{ m}$) from a neon lamp irradiates a photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54 V . The source is replaced by an iron source and its 427.2 nm line irradiates the same photocell. Predict the new stopping voltage.

Ans. Here $\lambda_1 = 640.2 \times 10^{-9} \text{ m}$, $V_{01} = 0.54 \text{ V}$,
 $\lambda_2 = 427.2 \times 10^{-9} \text{ m}$, $V_{02} = ?$

From Einstein's photoelectric equation,

$$eV_0 = \frac{hc}{\lambda} - W_0$$

For neon lamp,

$$eV_{01} = \frac{hc}{\lambda_1} - W_0$$

For iron source,

$$eV_{02} = \frac{hc}{\lambda_2} - W_0$$

$$\therefore V_{02} - V_{01} = \frac{hc}{e} \left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right]$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}}$$

$$\times \left[\frac{1}{427.2 \times 10^{-9}} - \frac{1}{640.2 \times 10^{-9}} \right]$$

$$= \frac{6.63 \times 3 \times 10^2}{1.6} \times \frac{213.0}{427.2 \times 640.2}$$

$$= 0.97 \text{ V}$$

$$\therefore V_{02} = V_{01} + 0.97$$

$$= 0.54 + 0.97 = 1.51 \text{ V}.$$

11.28. A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used :

$$\lambda_1 = 3650 \text{ \AA}, \quad \lambda_2 = 4047 \text{ \AA}, \quad \lambda_3 = 4358 \text{ \AA},$$

$$\lambda_4 = 5461 \text{ \AA}, \quad \lambda_5 = 6907 \text{ \AA}.$$

The stopping voltages, respectively, were measured to be :

$$V_{01} = 1.28 \text{ V}, \quad V_{02} = 0.95 \text{ V}, \quad V_{03} = 0.74 \text{ V},$$

$$V_{04} = 0.16 \text{ V}, \quad V_{05} = 0 \text{ V}$$

(a) Determine the value of Planck's constant h .

(b) Estimate the threshold frequency and work function for the material.

Ans. (a) Using $\nu = \frac{c}{\lambda}$, we first determine frequency in

each case and then plot a graph between stopping potential V_0 and frequency ν .

λ	ν	V_0
3650 \AA	$8.2 \times 10^{14} \text{ Hz}$	1.28 V
4047 \AA	$7.4 \times 10^{14} \text{ Hz}$	0.95 V
4358 \AA	$6.9 \times 10^{14} \text{ Hz}$	0.74 V
5461 \AA	$5.5 \times 10^{14} \text{ Hz}$	0.16 V
6907 \AA	$4.3 \times 10^{14} \text{ Hz}$	0.0 V

V_0 versus ν plot is shown in Fig. 11.30. The first four points lie nearly on a straight line which intercepts the ν -axis at threshold frequency $\nu_0 = 5.0 \times 10^{14}$ Hz. The fifth point $\nu (= 4.3 \times 10^{14}$ Hz) corresponds to $\nu < \nu_0$, so there is no photoelectric emission and no stopping voltage is required to stop the current.

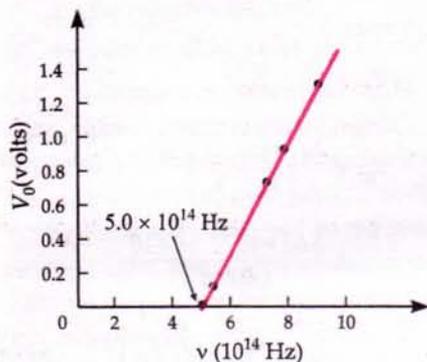


Fig. 11.30

Slope of V_0 versus ν graph is

$$\frac{\Delta V}{\Delta \nu} = \frac{(1.28 - 0) \text{ V}}{(8.2 - 5.0) \times 10^{14} \text{ s}^{-1}} = 4.0 \times 10^{-15} \text{ Vs}$$

From Einstein's photoelectric equation,

$$\text{K.E.} = eV = h\nu - W_0$$

$$\therefore e \Delta V = h \Delta \nu \quad [\because W_0 \text{ is a constant}]$$

or
$$\frac{\Delta V}{\Delta \nu} = \frac{h}{e}$$

Hence
$$\frac{h}{e} = 4.0 \times 10^{-15} \text{ Vs}$$

Planck's constant,

$$\begin{aligned} h &= e \times 4.0 \times 10^{-15} \text{ Js} \\ &= 1.6 \times 10^{-19} \times 4.0 \times 10^{-15} \text{ Js} \\ &= 6.4 \times 10^{-34} \text{ Js} \end{aligned}$$

(b) Threshold frequency, $\nu_0 = 5.0 \times 10^{14}$ Hz

\therefore Work function,

$$\begin{aligned} W_0 &= h\nu_0 = 6.4 \times 10^{-34} \times 5.0 \times 10^{14} \text{ J} \\ &= \frac{6.4 \times 5.0 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 2.00 \text{ eV.} \end{aligned}$$

11.29. The work function for the following metals is given :

Na : 2.75 eV ; K : 2.30 eV ; Mo : 4.17 eV ; Ni : 5.15 eV

Which of these metals will not give p.e. emission for a radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell ? What happens if the laser is brought nearer and placed 50 cm away ?

[CBSE OD 90 C]

Ans. Wavelength of incident radiation is

$$\lambda = 3300 \text{ Å} = 3300 \times 10^{-10} \text{ m}$$

Energy of an incident photon,

$$\begin{aligned} E &= \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3300 \times 10^{-10}} \text{ J} \\ &= \frac{6.63 \times 3 \times 10^{-18}}{33 \times 1.6 \times 10^{-19}} \text{ eV} \\ &= 3.75 \text{ eV} \end{aligned}$$

As the energy of incident photon is greater than the work functions of Na and K but less than the work functions of Mo and Ni, so the metals Mo and Ni will not give photoelectric emission.

If the laser is brought closer, intensity of radiation increases. This does not affect the result regarding Mo and Ni, but the photoelectric current will increase for Na and K with the increase in intensity.

11.30. Light of intensity 10^{-5} Wm^{-2} falls on a sodium photocell of surface area 2 cm^2 . Assuming that the top 5 layers of sodium absorb the incident energy, estimate the time required for photoelectric emission in the wave picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer ?

Ans. Suppose sodium has one conduction electron available per atom.

$$\text{Effective atomic area} \approx 10^{-20} \text{ m}^2$$

\therefore Number of conduction electrons in 5 layers

$$\begin{aligned} &= \frac{5 \times \text{Area of 1 layer}}{\text{Effective atomic area}} \\ &= \frac{5 \times 2 \times 10^{-4} \text{ m}^2}{10^{-20} \text{ m}^2} = 10^{17} \end{aligned}$$

$$\text{Now, Intensity} = \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{\text{Power}}{\text{Area}}$$

$$\begin{aligned} \therefore \text{Incident power} &= \text{Incident intensity} \times \text{area} \\ &= 10^{-5} \text{ Wm}^{-2} \times 2 \times 10^{-4} \text{ m}^2 \\ &= 2 \times 10^{-9} \text{ W} \end{aligned}$$

In terms of wave picture, incident power is uniformly absorbed by all the electrons continuously.

\therefore Energy absorbed per second (or power) per electron

$$= \frac{2 \times 10^{-9}}{10^{17}} = 2 \times 10^{-26} \text{ W}$$

Time required for photoelectric emission

$$\begin{aligned} &= \frac{\text{Energy required per electron}}{\text{Energy absorbed per second per electron}} \\ &= \frac{2 \text{ eV}}{2 \times 10^{-26} \text{ W}} = \frac{2 \times 1.6 \times 10^{-19} \text{ J}}{2 \times 10^{-26} \text{ Js}^{-1}} \\ &= 1.6 \times 10^7 \text{ s} \approx 0.5 \text{ year.} \quad [\text{Given } W_0 = 2 \text{ eV}] \end{aligned}$$

Implication. Experimentally, photoelectric emission is observed nearly instantaneously ($\approx 10^{-9}$ s). Thus the wave picture is in gross disagreement with experiment.

11.31. Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy – an X-ray photon or the electron? (For quantitative comparison, take the wavelength of the probe equal to 1 \AA , which is of the order of interatomic spacing in the lattice) ($m_e = 9.11 \times 10^{-31} \text{ kg}$).

Ans. For X-rays photons: $\lambda = 1 \text{ \AA} = 10^{-10} \text{ m}$

Energy of a photon is

$$\begin{aligned} E &= h\nu = \frac{hc}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-10}} \text{ J} \\ &= \frac{6.63 \times 3 \times 10^{-16}}{1.6 \times 10^{-19}} \text{ eV} \\ &= 12.4 \times 10^3 \text{ eV} = \mathbf{12.4 \text{ keV}}. \end{aligned}$$

For electrons:

$$\lambda = 1 \text{ \AA} = 10^{-10} \text{ m}, \quad m = 9.11 \times 10^{-31} \text{ kg}$$

\therefore Momentum,

$$\begin{aligned} p &= \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-10}} \\ &= 6.63 \times 10^{-24} \text{ kg ms}^{-1} \end{aligned}$$

Energy of an electron is

$$\begin{aligned} K &= \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31}} \text{ J} \\ &= \frac{6.63 \times 6.63 \times 10^{-17}}{2 \times 9.11 \times 1.6 \times 10^{-19}} \text{ eV} \quad [\because p = mv] \\ &= \mathbf{150.6 \text{ eV}} \end{aligned}$$

Thus for the same wavelength, a photon has much greater energy than an electron.

11.32. (a) Obtain the de-Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in Exercise 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain. ($m_n = 1.675 \times 10^{-27} \text{ kg}$). (b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27°C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Ans. (a) Here $m_n = 1.675 \times 10^{-27} \text{ kg}$

$$\begin{aligned} K &= 150 \text{ eV} = 150 \times 1.6 \times 10^{-19} \text{ J} \\ &= 2.4 \times 10^{-17} \text{ J} \end{aligned}$$

$$\text{As} \quad K = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

$$\text{or} \quad p = \sqrt{2mK}$$

\therefore de-Broglie wavelength of neutrons is

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{h}{\sqrt{2mK}} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 2.4 \times 10^{-17}}} \\ &= \frac{6.63 \times 10^{-11}}{28.35} \text{ m} = \mathbf{2.33 \times 10^{-12} \text{ m}} \end{aligned}$$

As the interatomic spacing ($1 \text{ \AA} = 10^{-10} \text{ m}$) is about hundred times greater than this wavelength, so a neutron beam of 150 eV energy is not suitable for diffraction experiments.

(b) Average kinetic energy of a neutron at absolute temperature T is

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{3}{2}kT \\ \text{or} \quad \frac{p^2}{2m} &= \frac{3}{2}kT & [\because p = mv] \\ \text{or} \quad p &= \sqrt{3mkT} \end{aligned}$$

\therefore de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

Given $m_n = 1.675 \times 10^{-27} \text{ kg}$,

$$k = 1.38 \times 10^{-23} \text{ J mol}^{-1} \text{K}^{-1}$$

$$T = 27 + 273 = 300 \text{ K}, \quad h = 6.63 \times 10^{-34} \text{ Js}$$

$$\begin{aligned} \lambda &= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \\ &= \frac{6.63 \times 10^{-10}}{4.56} \text{ m} \\ &= \mathbf{1.45 \times 10^{-10} \text{ m} = 1.45 \text{ \AA}} \end{aligned}$$

As this wavelength is comparable to interatomic spacing ($\approx 1 \text{ \AA}$) in a crystal, so thermal neutrons can be used for diffraction experiments. A high energy neutron beam should be first thermalised before using it for diffraction.

11.33. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. If other factors (such as numerical aperture etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

[CBSE OD 14]

Ans. Here $V = 50 \text{ kV} = 5 \times 10^4 \text{ V}$, $m_e = 9.11 \times 10^{-31} \text{ kg}$

K.E. of an electron,

$$\begin{aligned} K &= 5 \times 10^4 \text{ eV} \\ &= 1.6 \times 10^{-19} \times 5 \times 10^4 \text{ J} = \mathbf{8 \times 10^{-15} \text{ J}} \end{aligned}$$

∴ de-Broglie wavelength of electrons is

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{2mK}} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 8 \times 10^{-15}}} \text{ m} \\ &= \frac{6.63 \times 10^{-11}}{12.07} \text{ m} = 5.5 \times 10^{-12} \text{ m}\end{aligned}$$

Wavelength of yellow light,

$$\lambda_y = 5.9 \times 10^{-7} \text{ m}$$

Resolving power of a microscope $\propto \frac{1}{\lambda}$

$$\begin{aligned}\therefore \frac{\text{Resolving power of electron microscope}}{\text{Resolving power of optical microscope}} &= \frac{\lambda_y}{\lambda} \\ &= \frac{5.9 \times 10^{-7}}{5.5 \times 10^{-12}} = 10^5\end{aligned}$$

Thus, the resolving power of an electron microscope is about 10^5 times greater than that of an optical microscope.

11.34. The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length scale of 10^{-15} m or less. This structure was first probed in early 1970's using high energy electron beams produced by a Linear Accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV).

Ans. Here $\lambda = 10^{-15}$ m, $h = 6.63 \times 10^{-34}$ Js

∴ Momentum,

$$\begin{aligned}p &= \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ Js}}{10^{-15} \text{ m}} \\ &= 6.63 \times 10^{-19} \text{ kg ms}^{-1}\end{aligned}$$

Rest mass energy of electron,

$$\begin{aligned}m_0 c^2 &= 0.511 \text{ MeV} \\ &= 0.511 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} \\ &= 0.511 \times 1.6 \times 10^{-13} \text{ J}.\end{aligned}$$

We use the relativistic formula for the energy of electron, i.e.,

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

$$\begin{aligned}\therefore E^2 &= p^2 c^2 + m_0^2 c^4 \\ &= (6.63 \times 10^{-19})^2 \times (3 \times 10^8)^2 \\ &\quad + (0.511 \times 1.6 \times 10^{-13})^2 \text{ J} \\ &= (6.63)^2 \times 9 \times 10^{-22} + (0.511 \times 1.6)^2 \times 10^{-26} \text{ J} \\ &= (6.63)^2 \times 9 \times 10^{-22}.\end{aligned}$$

The rest-mass energy (second) term being negligible, therefore,

$$\begin{aligned}E &= 6.63 \times 3 \times 10^{-11} \text{ J} \\ &= \frac{1.989 \times 10^{-10}}{1.6 \times 10^{-10}} \text{ BeV} \quad (\because 1 \text{ BeV} = 1.6 \times 10^{-10} \text{ J}) \\ &= 1.24 \text{ BeV}\end{aligned}$$

Thus the electron energies from the accelerator must have been of the order of a few BeV.

11.35. Find the typical de-Broglie wavelength associated with a He atom in helium gas at room temperature (27°C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.

Ans. Mass of the atom is given by

$$\begin{aligned}m &= \frac{\text{Atomic wt. of He}}{\text{Avogadro's number}} \\ &= \frac{4}{6 \times 10^{23}} \text{ g} = \frac{4 \times 10^{-3}}{6 \times 10^{23}} \text{ kg} \\ &= \frac{2}{3} \times 10^{-26} \text{ kg}\end{aligned}$$

$$T = 27 + 273 = 300 \text{ K}$$

Average K.E. of a He atom at absolute temperature T is

$$\frac{1}{2} m v^2 = \frac{3}{2} kT$$

$$\therefore m^2 v^2 = 3mkT$$

$$\text{or } p^2 = 3mkT$$

$$p = \sqrt{3mkT}$$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

$$\begin{aligned}&= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times \frac{2}{3} \times 10^{-26} \times 1.38 \times 10^{-23} \times 300}} \\ &= 0.73 \times 10^{-10} \text{ m}.\end{aligned}$$

Kinetic gas equation for one mole of a gas can be written as

$$PV = RT$$

$$\text{or } PV = kNT$$

$$\left[\because k = \frac{R}{N} \right]$$

$$\text{or } \frac{V}{N} = \frac{kT}{P}$$

∴ Mean separation,

$$\begin{aligned}r &= \left[\frac{\text{Molar Volume}}{\text{Avogadro's Number}} \right]^{1/3} \\ &= \left[\frac{kT}{P} \right]^{1/3}\end{aligned}$$

Given $T = 300 \text{ K}$, $P = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$

$$k = 1.38 \times 10^{-23} \text{ J mol}^{-1} \text{ K}^{-1}$$

$$\begin{aligned} \text{Hence } r &= \left[\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5} \right]^{1/3} \text{ m} \\ &= \left[\frac{138 \times 30}{101} \right]^{-1/3} \times 10^{-9} \text{ m} \\ &= 3.4 \times 10^{-9} \text{ m}. \end{aligned}$$

We find that $r \gg \lambda$, i.e., the wave packets associated with He atoms do not overlap and hence He atoms can be distinctly seen.

11.36. Find the typical de-Broglie wavelength of an electron in a metal at 27°C and compare it with the mean separation between two electrons in a metal which is given to be about $2 \times 10^{-10} \text{ m}$.

Ans. Mass of an electron is

$$m = 9.11 \times 10^{-31} \text{ kg}$$

and $T = 27 + 273 = 300 \text{ K}$

\therefore de-Broglie wavelength of electrons is

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{3mkT}} \\ &= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}} \text{ m} \\ &= \frac{6.63 \times 10^{-8}}{\sqrt{3 \times 9.11 \times 1.38 \times 3}} = \frac{6.63 \times 10^{-8}}{10.64} \\ &= 6.2 \times 10^{-9} \text{ m} \end{aligned}$$

Mean separation between two electrons in a metal is

$$r = 2 \times 10^{-10} \text{ m}$$

$$\therefore \frac{\lambda}{r} = \frac{6.2 \times 10^{-9}}{2 \times 10^{-10}} = 31$$

Thus the de-Broglie wavelength is much greater than the given inter-electron separation.

11.37. Answer the following questions :

- Quarks inside protons and neutrons are thought to carry fractional charges $\left(+\frac{2}{3}e, -\frac{1}{3}e \right)$. Why do they not show up in Millikan's oil drop experiment ?
- Why do we need the oil drops of Millikan's experiment to be of such microscopic sizes ? Why cannot we experiment with much bigger drops ?
- Stoke's formula for viscous drag is not really valid for oil drops of extremely minute sizes. Why not ?
- Every metal has a definite work function. Why do photoelectrons not come out all with same energy if incident radiation is monochromatic ? Why is there an energy distribution of photoelectrons ?

(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations :

$$E = h\nu, \quad p = \frac{h}{\lambda}$$

But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed $\nu\lambda$) has no physical significance. Why ?

Ans. (a) Quarks are thought to be confined within a proton or neutron by forces which grow stronger if one tries to pull them apart. It, therefore, seems that though fractional charges may exist in nature, observable charges are still integral multiples of electronic charge e .

(b) Electric fields needed in the experiment with much bigger drops will be impractically high.

(c) Stokes' formula is valid for motion through a homogeneous continuous medium. The size of the drop should be much larger than the intermolecular separation in the medium for this assumption to be valid; otherwise the drop 'sees' inhomogeneties in the medium, i.e., there is concentrated mass density in the molecules, and voids in between the molecules.

(d) Work-function merely indicates the minimum energy required for the electrons in the highest level of the conduction band to get out of the metal. Not all electrons in the metal belong to this level. They occupy a continuous band of levels. Consequently, for the same incident radiation, electrons knocked off from different levels come out with different energies.

(e) The de Broglie wavelength, $\lambda = h/p$ of the matter wave of an electron has a fixed value and λ has physical significance. But the absolute value of the energy E of any particle is arbitrary to within an additive constant. Consequently, absolute value of frequency ν of a matter wave of an electron has no direct physical meaning. Likewise, the phase speed $v_p = \nu\lambda$ is not physically significant. However, the group speed of the matter wave is physically significant and equals the speed of the particle as proved below.

$$\text{As } v_p = \nu\lambda = \frac{\omega}{k}$$

\therefore Group speed

$$\begin{aligned} &= \frac{d\omega}{dk} = \frac{d\nu}{d(1/\lambda)} \\ &= \frac{d(h\nu)}{d(h/\lambda)} \\ &= \frac{dE}{dp} = \frac{d}{dp} \left(\frac{p^2}{2m} \right) \\ &= \frac{p}{m} = \text{particle speed.} \end{aligned}$$

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. Define work function for a given metallic surface. [Himachal 04 ; CBSE D 03, 04C]
2. On what factors does the work function of a metal depend ?
3. What is thermionic emission ?
4. Mention one physical process for the release of electrons from a metal surface. [Haryana 02]
5. Name a phenomenon which illustrates particle nature of light. [Haryana 02]
6. What is a photon ?
7. Name one factor on which the energy of one quantum of light depends. [Punjab 99]
8. What is the rest mass of photons ? [Punjab 99C]
9. Define intensity of radiation on the basis of photon picture of light. Write its SI unit. [CBSE OD 14]
10. What is the momentum of a photon of frequency ν ? [Himachal 04]
11. A photon has velocity c and frequency ν . Write down the expression for the wavelength. [CBSE F 04]
12. Write down the relation between the energy and momentum of a photon. [ISCE 93]
13. Write down the relation between wavelength and momentum of a photon ? What do the symbols stand for ? [ISCE 01]
14. Define electron volt. [Punjab 99]
15. How many electron volts make up one joule ? [Himachal 93]
16. What is photoelectric effect ? [ISCE 02]
17. What are photoelectrons ?
18. Can we use any substance as photo-electron emitter ?
19. Define the term 'threshold frequency' as used in photoelectric effect. [CBSE OD 14C]
20. Write Einstein's photoelectric equation. [Haryana 02 ; ISCE 01]
21. How is work function related to threshold frequency ?
22. What is threshold wavelength for photoelectric effect ?
23. Define the term 'stopping potential' in relation to photoelectric effect. [CBSE OD 11, 14C]
24. If the intensity of the incident radiation in a photo-cell is increased, how does the stopping potential vary ? [CBSE D 03]
25. If the intensity of incident radiation on a metal is doubled, what happens to the kinetic energy of electrons emitted ? [CBSE OD 93]
26. Does the stopping potential depend on the frequency of incident radiation ?
27. Threshold wavelength for photoelectric emission is 5000 \AA . Will the photoelectrons be emitted when this material is illuminated with monochromatic radiation from 1 watt ultraviolet lamp ?
28. What determines the maximum velocity of photoelectrons ?
29. Two metals A, B have work functions 2 eV and 4 eV respectively. Which metal has a lower threshold wavelength for photoelectric effect ? [Haryana 02 ; CBSE D 02]
30. Light of frequency 1.5 times the threshold frequency is incident on a photosensitive material. If the frequency is halved and intensity is doubled, what happens to photoelectric current ?
31. Two metals X and Y when illuminated with appropriate radiation emit photoelectrons. The work function of X is higher than that of Y . Which metal will have higher threshold frequency and why ? [CBSE OD 01C]
32. Show graphically how the stopping potential for a given photo-sensitive surface varies with the frequency of the incident radiation. [CBSE OD 05]
33. A graph is plotted between the maximum K.E. of emitted photo-electrons and the frequency of incident radiations. Which physical constant can be determined from slope of this graph ? [CBSE Sample Paper 03]
34. Plot a graph showing the variation of photoelectric current as a function of anode potential for two light beams having the same frequency but different intensities I_1 and $I_2 (I_1 > I_2)$. [CBSE OD 14C]
35. In an experiment on photoelectric effect, the following graphs were obtained between the photoelectric current (I) and the anode potential (V). Name the characteristic of the incident radiation that was kept constant in this experiment.

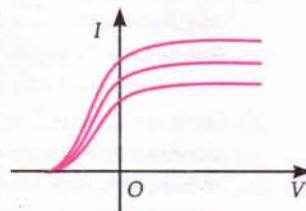


Fig. 11.31

36. Calculate the work function of a metal in eV if its threshold wavelength is 6800 \AA and $h = 6.62 \times 10^{-27} \text{ erg s}$. [Haryana 96]
37. Work function of Na is 2.3 eV. Does sodium show photoelectric emission for light of wavelength 6800 \AA ? [Haryana 94]
38. Is there any difference between light waves and matter waves?
39. What is de-Broglie wavelength?
40. An electron is accelerated through a potential difference of 300 V. What is its energy in electron volt? [Himachal 04]
41. Write the expression for the de-Broglie wavelength associated with a charged particle having charge ' q ' and mass ' m ', when it is accelerated by a potential V . [CBSE OD 13]
42. Name the experiment for which the following graph, showing the variation of intensity of scattered electrons with the angle of scattering, was obtained. Also name the important hypothesis that was confirmed by this experiment. [CBSE OD 04C]

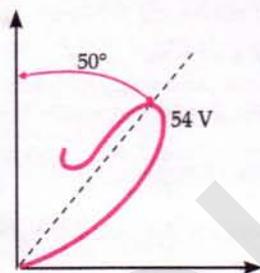


Fig. 11.32

43. What information is derived from electron diffraction experiments? [ISCE 97]
44. What is the momentum p of a photon from ultraviolet light of wavelength $\lambda = 332 \text{ nm}$? [ISCE 03]
45. What is the role of a photo-cell in cinematography?
46. How will the photoelectric current change on decreasing the wavelength of incident radiation for a given photosensitive material? [CBSE D 06C]
47. Name the experiment which establishes the wave nature of a particle. [CBSE OD 06C]
48. With what purpose was famous Davisson-Germer experiment with electrons performed? [CBSE D 06]
49. de Broglie wavelength associated with an electron accelerated through a potential difference V is λ . What will be its wavelength when the accelerating potential is increased to 4 V? [CBSE OD 06]

50. What is the de Broglie wavelength (in \AA) associated with an electron accelerated through a potential of 100 V? [CBSE F 06]
51. What is the stopping potential of a photocell, in which electrons with a maximum kinetic energy of 6 eV are emitted? [CBSE OD 08]
52. How does the stopping potential applied to a photocell change, if the distance between the light source and the cathode of the cell is doubled? [CBSE OD 08]
53. The given graphs show the variation of photoelectric current (I) with the applied voltage (V) for two different materials and for two different intensities of the incident radiation. Identify the pairs of curves that correspond to different materials but same intensity of incident radiation. [CBSE D 13]

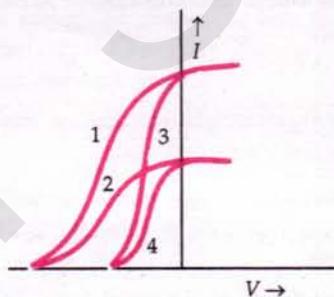


Fig. 11.33

54. Write the relationship of de-Broglie wavelength λ associated with a particle of mass m in terms of its kinetic energy K . [CBSE D 11C]
55. Figure 11.34 shows the plot of $\frac{1}{\sqrt{V}}$ (V is the accelerating potential) vs. the de-Broglie wavelength λ for two particles having the same charge. Which of the two lines A or B represents a particle of larger mass? [CBSE OD 13C]

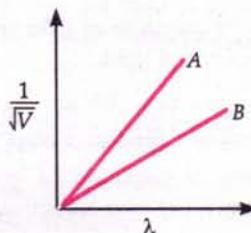


Fig. 11.34

56. Show on a graph the variation of de-Broglie wavelength λ associated with an electron, with the square root of the accelerating potential. [CBSE F 12]

57. Show, on a graph, the nature of variation, of the (associated) de-Broglie wavelength (λ), with the accelerating potential (V), for an electron initially at rest. [CBSE D 11]
58. A proton, and an alpha particle, both initially at rest, are (suitably) accelerated so as to have the same kinetic energy. What is the ratio of their de-Broglie wavelengths? [CBSE Sample Paper 11]
59. A proton and an electron have same kinetic energy. Which one has greater de-Broglie wavelength and why? [CBSE OD 12]
60. The de-Broglie wavelengths, associated with a proton and a neutron, are found to be equal. Which of the two has a higher value for kinetic energy? [CBSE Sample Paper 08]
61. An electron and alpha particle have the same de-Broglie wavelength associated with them. How are their kinetic energies related to each other? [CBSE D 08]
62. An electron and alpha particle have the same kinetic energy. How are the de-Broglie wavelengths associated with them related? [CBSE D 08]
63. An electron and a proton have the same de-Broglie wavelength associated with them. How are their kinetic energies related to each other? [CBSE D 08]
64. Show on a plot the nature of variation of photoelectric current with the intensity of radiation incident on a photosensitive surface. [CBSE D 13C, 14]
65. Find the ratio of de-Broglie wavelengths associated with two electron beams accelerated through 25 V and 36 V respectively. [CBSE OD 13C]
66. Draw a plot showing the variation of de-Broglie wavelength of electron as a function of its K.E. [CBSE D 15C]

Answers

1. The minimum energy needed to pull an electron from a metal surface is called the work function of the metal.
2. The work function of a metal depends on its nature and the conditions of its surface.
3. When a metal is heated, its free electrons get thermal energy more than the work function of the metal and get ejected from the metal surface. This process is called thermionic emission.
4. Photoelectron emission. In this process, electrons are emitted from a metal surface when it is exposed to e.m. radiations of a suitable frequency.
5. Photoelectric effect.
6. One quantum of electromagnetic radiation is called a photon.
7. The energy of quantum of light (photon) depends on the frequency of light.
 $E = h\nu$ i.e., $E \propto \nu$.
8. The rest mass of a photon is zero.
9. The intensity of radiation of given wavelength represents the number of energy quanta or photons incident per unit area per unit time, with each photon having the same energy.
 Intensity of radiation = $\frac{\text{Energy}}{\text{Area} \times \text{Time}}$
 \therefore SI unit of intensity = $\text{Js}^{-1}\text{m}^{-2}$ or Wm^{-2} .
10. Energy, $E = h\nu = mc^2$
 Momentum, $p = mc = \frac{h\nu}{c}$.
11. Wavelength, $\lambda = \frac{c}{\nu}$.
12. $p = \frac{E}{c}$, where c is the speed of light.
13. Momentum = $\frac{\text{Planck's constant}}{\text{Wavelength}}$
 or $p = \frac{h}{\lambda}$.
14. The energy gained by an electron, when accelerated through a potential difference of 1 volt, is called electron volt (eV).
 $1\text{eV} = 1.6 \times 10^{-19}\text{J}$
15. $1\text{J} = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}\text{eV}$.
16. Photoelectric effect is the phenomenon of emission of electrons from a metal surface when it is exposed to e.m. radiations of a suitable frequency.
17. These are the electrons emitted from a metal surface when it is exposed to e.m. radiations of a suitable frequency.
18. Yes, any substance can be made to emit photoelectrons by exposing it to electromagnetic radiations of sufficiently high frequency.

19. The minimum value of the frequency of incident radiation below which the photoelectric emission stops altogether is called threshold frequency. The threshold frequency depends on the nature of photoelectric emitter.

20. According to Einstein's photoelectric equation, the maximum kinetic energy of a photoelectron is given by

$$K = \frac{1}{2} m v_{\max}^2 = h\nu - W_0$$

where ν = frequency of incident radiation,
and W_0 = work function of the metal.

21. $h\nu_0 = W_0$,

where ν_0 = threshold frequency and
 W_0 = work function.

22. The maximum wavelength of incident radiation above which photoelectric emission does not occur is called threshold wavelength for the given metal surface.

23. The minimum negative potential given to the anode of a photo-cell for which the photoelectric current becomes zero is called stopping potential.

24. Stopping potential remains unaffected by the increase in the intensity of incident radiation.

25. Kinetic energy of photo-electrons remains unaffected. It does not depend on the intensity of incident radiation.

26. Yes, the stopping potential V_0 depends on the frequency ν of the incident radiation.

$$V_0 = \frac{h}{e} (\nu - \nu_0)$$

27. Yes, because wavelength of ultraviolet radiation is less than that of threshold wavelength 5000 Å.

28. It depends on the frequency of incident radiation and work function of the metal.

29. Work function,

$$W_0 = h\nu_0 = \frac{hc}{\lambda_0}$$

$$\therefore \lambda_0 \propto \frac{1}{W_0}$$

As metal B has higher work function, so it has lower threshold wavelength.

30. Photoelectric current becomes zero, because the new frequency is less than the threshold frequency.

31. Work function, $W_0 = h\nu_0$. As metal X has higher work function, so it has a higher threshold frequency.

32. See Fig. 11.7 on page 11.4.

33. Planck's constant h can be determined from the slope of this graph.

34. See Fig. 11.5 on page 11.4.

35. The frequency of the incident radiation is kept constant in the experiment.

36. Here $\lambda_0 = 6800 \text{ \AA} = 6.8 \times 10^{-7} \text{ m}$,

$$h = 6.62 \times 10^{-27} \text{ erg s} = 6.62 \times 10^{-34} \text{ Js}$$

$$W_0 = \frac{hc}{\lambda_0} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{6.8 \times 10^{-7}} \\ = 2.92 \times 10^{-19} \text{ J} = 1.825 \text{ eV.}$$

37. Energy of photon

$$= \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{6800 \times 10^{-10}} \\ = 2.92 \times 10^{-19} \text{ J} = 1.825 \text{ eV}$$

As the energy of photon is less than the work function ($W_0 = 2.3 \text{ eV}$) of Na, so photoelectric emission does not occur.

38. Yes, light waves are electromagnetic in nature while matter waves are associated with moving particles.

39. The wavelength of the waves associated with a beam of moving particles is called de-Broglie wavelength.

40. 300 eV.

$$41. \quad qV = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$$\Rightarrow p = \sqrt{2mqV}$$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

42. Davisson-Germer experiment. This experiment confirmed the de Broglie hypothesis of matter waves.

43. This experiment proves the existence of electron waves and hence confirms the de Broglie hypothesis of matter waves.

$$44. \quad p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ Js}}{332 \times 10^{-9} \text{ m}} = 2.0 \times 10^{-27} \text{ kg ms}^{-1}.$$

45. In cinematography, photocells are used for the reproduction of sound.

46. Photoelectric current is not affected on decreasing the wavelength of incident radiation.

47. Davisson and Germer experiment.

48. Davisson-Germer experiment confirmed the wave nature of electrons.

49. de-Broglie wavelength,

$$\lambda \propto \frac{1}{\sqrt{V}}$$

$$\therefore \frac{\lambda'}{\lambda} = \sqrt{\frac{V}{4V}} = \frac{1}{2}$$

or $\lambda' = \lambda/2$

$$50. \lambda = \frac{12.27}{\sqrt{V}} \text{ \AA} = \frac{12.27}{\sqrt{100}} \text{ \AA} \\ = 1.227 \text{ \AA}.$$

51. Stopping potential,

$$V_0 = \frac{K_{\max}}{e} = \frac{6 \text{ eV}}{e} = 6 \text{ V}.$$

52. The stopping potential remains same. When the distance is doubled, the intensity of incident radiation becomes one-fourth of the original intensity. But the stopping potential is independent of intensity.

53. (i) Curves 1 and 3.

(ii) Curves 2 and 4.

$$54. K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

or $p = \sqrt{2mK}$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$$55. \lambda = \frac{h}{\sqrt{2mqV}}$$

or $\frac{1/\sqrt{V}}{\lambda} = \frac{\sqrt{2mq}}{h}$

\therefore Slope $\propto \sqrt{m}$ [$\because q$ is same]

Hence, the line *A* with larger slope represents the particle of larger mass.

56. As $\lambda \propto \frac{1}{\sqrt{V}}$, the graph between λ and \sqrt{V} is a hyperbola as shown in Fig. 11.35.

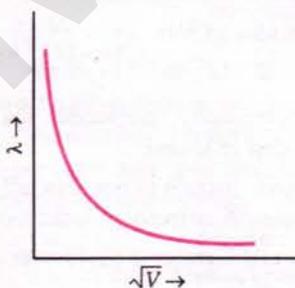


Fig. 11.35

57. As $\lambda \propto \frac{1}{\sqrt{V}}$, so the graph between de-Broglie wavelength (λ) and accelerating potential (V) is of the type as shown in Fig. 11.36.

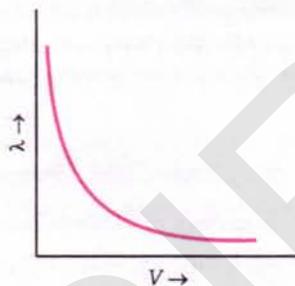


Fig. 11.36

58. de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \quad \text{i.e., } \lambda \propto \frac{1}{\sqrt{m}}$$

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}} = \sqrt{\frac{4m_p}{m_p}} = 2 : 1.$$

$$59. \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \quad \text{i.e., } \lambda \propto \frac{1}{\sqrt{m}}$$

As $m_e \ll m_p$, so $\lambda_e \gg \lambda_p$

Hence protons have greater de-Broglie wavelength.

$$60. \lambda = \frac{h}{\sqrt{2mK}}$$

As $\lambda_p = \lambda_n$

or $\frac{h}{\sqrt{2m_p K_p}} = \frac{h}{\sqrt{2m_n K_n}}$

or $m_p K_p = m_n K_n$

But $m_p < m_n \therefore K_p > K_n$

Thus the proton has a higher value of kinetic energy.

$$61. K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \quad \text{i.e., } K \propto \frac{1}{m}$$

$$\therefore \frac{K_e}{K_\alpha} = \frac{m_\alpha}{m_e}$$

$$62. K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \quad \text{or} \quad \lambda = \frac{h}{\sqrt{2mK}}$$

i.e., $\lambda \propto \frac{1}{\sqrt{m}}$

$$\therefore \frac{\lambda_e}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_e}}$$

$$63. K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \text{ i.e., } K = \frac{1}{m}$$

$$\therefore \frac{K_e}{K_p} = \frac{m_p}{m_e}$$

64. See Fig. 11.4 on page 11.3.

$$65. \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{36}{25}} = 6:5.$$

$$66. \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \Rightarrow \lambda^2 \propto \frac{1}{K}$$

The graph of λ vs. K.E. is of the type shown below.

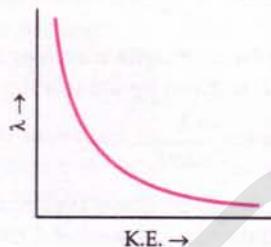


Fig. 11.37

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

- What is photo-electric effect? State the laws of photo-electric emission. [Haryana 10; Punjab 11]
- What is photo-electric effect? Why it cannot be explained on the basis of wave nature of light? [CBSE D 94]
- What is photo-electric effect? Explain the effect of increase of (i) frequency (ii) intensity of the incident radiation on the photo-electrons emitted by a phototube. [CBSE OD 94, 95C, 96C]
- With reference to the photo-electric effect, define the terms 'work-function' and 'threshold wavelength' for a given metal. On what factors will the following depend during photo-electric emission from a metal surface :
(i) the magnitude of photo-electric current, and
(ii) the velocity of ejected electrons? [ISCE 98]
- Explain 'stopping potential' and 'threshold frequency' in photo-electric emission. Give an appropriate graph between them. [ISCE 96]
- Obtain the expression for the maximum kinetic energy of the electrons emitted from a metal surface in terms of the frequency of the incident radiation and the threshold frequency. [CBSE D 97C]
- Explain the laws of photoelectric emission on the basis of Einstein's photoelectric equation. Write one feature of the photo-electric effect which cannot be explained on the basis of wave theory of light. [CBSE OD 98, F 06]
- Draw the graph showing the variation of photo-electric current with anode potential of a photocell for
(i) the same frequencies but different intensities $I_3 > I_2 > I_1$ of incident radiation, and
(ii) the same intensity but different frequencies $\nu_1 > \nu_2 > \nu_3$ of incident radiation. [CBSE OD 06C]
- Draw a graph to show the variation of stopping potential with frequency of radiations incident on a metal plate. How can the value of Planck's constant be determined from this graph?
- Draw graph showing the variation of stopping potential with frequency of incident radiations in relation to photo-electric effect. Deduce an expression for the slope of this graph using Einstein's photo-electric equation. [CBSE OD 97]
- Define the terms 'work function' and 'threshold frequency' for photo-electric effect. Show graphically how stopping potential for a given metal varies with frequency of incident radiation. What does the slope of this graph represent? [CBSE D 98]
- (a) Why photoelectric effect cannot be explained on the basis of wave nature of light? Give reasons.
(b) Write the basic features of photon picture of electromagnetic radiation on which Einstein's photoelectric equation is based. [CBSE D 13]
- Write Einstein's photoelectric equation and point out any two characteristic properties of photons on which this equation is based. Briefly explain the three observed features which can be explained by this equation. [CBSE OD 13]
- (a) Write the important properties of photons which are used to establish Einstein's photoelectric equation.
(b) Use this equation to explain the concept of (i) threshold frequency and (ii) stopping potential. [CBSE OD 15]
- Explain the function of a photo-cell. Give its two uses. [ISCE 94]
- Derive de-Broglie wave equation for material particles.
- An electron and a proton have the same kinetic energy. Which of the two has greater wavelength? Justify your answer. [CBSE D 03C]

18. Describe briefly how wave nature of moving electrons was established experimentally.

[CBSE OD 15C]

19. Show that the de Broglie wavelength λ of electrons of energy K is given by the relation :

$$\lambda = \frac{h}{\sqrt{2mK}} \quad \text{[CBSE OD 93]}$$

20. Show that the de-Broglie wavelength λ of electrons accelerated through a potential difference of V volts can be expressed as

[CBSE D 92]

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{12.3}{\sqrt{V}} \text{ \AA}$$

21. In Davisson and Germer experiment, state the observations which led to

(i) show the wave nature of electrons, and

(ii) confirm the de-Broglie relation. [CBSE D 08C]

22. Write Einstein's photoelectric equation in terms of the stopping potential and the threshold frequency for a given photosensitive material. Draw a plot showing the variation of stopping potential vs. the frequency of incident radiation. [CBSE OD 08C]

23. Plot a graph showing variation of stopping potential (V_0) with the frequency (ν) of the incident radiation for a given photosensitive material. Hence state the significance of the threshold frequency in photoelectric emission.

Using the principle of energy conservation, write the equation relating the energy of incident photon, threshold frequency and the maximum kinetic energy of the emitted photoelectrons. [CBSE D 09C]

24. (a) Draw a graph showing variation of photoelectric current (I) with anode potential (V) for different intensities of incident radiation. Name the characteristic of the incident radiation that is kept constant in this experiment.

(b) If the potential difference used to accelerate electrons is doubled, by what factor does the de-Broglie wavelength associated with the electrons change? [CBSE F 09]

25. Derive an expression for the de Broglie wavelength associated with an electron accelerated through a potential V . Draw a schematic diagram of a localised-wave nature of the moving electron.

[CBSE F 09]

Answers

- Refer to point 7 and point 11 of Glimpses on pages 11.61 and 11.62.
- Refer to point 2 on page 11.61 and point 12 on page 11.62 of Glimpses .
- Refer answer to Q. 4 on page 11.3.
- Refer to points 2 and 10 of Glimpses on pages 11.61 and 11.62.
 - The magnitude of photo-current depends on the intensity of incident radiation.
 - The velocity of ejected electron depends on the frequency of incident radiation and work function of the metal.
- Refer to points 9 and 10 of Glimpses. See Fig. 11.7 on page 11.4.
- Refer to solution of Problem 3 on page 11.28.
- Refer to solution of Problem 3 on page 11.28. The wave theory cannot explain the existence of threshold frequency.
- See Fig. 11.6 and Fig. 11.7 on page 11.4.
- Refer answer to Q. 9 on page 11.7.
- Refer answer to Q. 9 on page 11.7.
- Refer to points 2 and 10 of Glimpses. See Fig. 11.9 on page 11.7. Slope of V_0 - ν graph = h/e .
- (a) Refer to point 12 of Glimpses on page 11.62.

(b) The basic features of the photon (or particle) picture of e.m. radiation are as follows :

(i) Light is composed of discrete packets of energy called quanta or photons.

(ii) Each photon carries an energy $E(=h\nu)$ and momentum $p(=h/\lambda)$, which depend on frequency ν of the incident radiation and not on its intensity.

(iii) Photoelectric emission from the metal surface occurs due to the absorption of a photon by an electron.

13. Refer to the solution of Problem 3 on page 11.28. For characteristic properties of photons, refer answer to the above question.

14. (a) Refer answer to Q. 12 (b) above.

(b) (i) $K_{\max} = h\nu - W_0 = h(\nu - \nu_0)$

For $\nu_0 < \nu$, K_{\max} will be negative.

Hence, for every metal there is a certain threshold frequency below which there is no photoelectric emission.

(ii) $K_{\max} = eV_0 = h(\nu - \nu_0)$

At $\nu = \nu_0$, $K_{\max} = eV_0 = 0$

No photoelectron is emitted at a certain negative value of anode potential. This potential is called stopping potential.

15. Refer answer to Q. 11 and Q. 12 on page 11.17.
16. Refer answer to Q. 14 on page 11.18.
17. Refer answer to Q. 15 on page 11.20.
18. Refer answer to Q. 16 on page 11.20.
19. Refer answer to Q. 15 on page 11.20.
20. Refer answer to Q. 15 on page 11.20.
21. (i) Intensity of scattered electrons depends on the scattering angle ϕ and always shows a bump for $\phi = 50^\circ$.
- (ii) de-Broglie wavelength determined from the experiment agrees with the value determined from the relation, $\lambda = \frac{h}{p} = \frac{1227}{\sqrt{V}}$ nm.
22. Refer answer to Q. 9 on page 11.7.
23. See Fig. 11.18 on page 11.29. For $v < v_0$, the stopping potential is zero. This indicates there is no photoelectric emission for $v < v_0$.
For derivation of Einstein's equation, refer to the solution of Problem 3 on page 11.28.
24. (a) See Fig. 11.5. Here frequency of the incident radiation is kept constant.
- (b) de-Broglie wavelength of electrons, $\lambda = \frac{12.3}{\sqrt{V}}$ Å
- When potential difference V is doubled, the wavelength becomes $1/\sqrt{2}$ times the original wavelength.
25. Refer answer to Q.15 on page 11.20 and see Fig. 11.13(a) on page 11.19.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. What is electron emission? Discuss various forms of electron emission. [Punjab 02]
2. Describe suitable experiments to study the laws of photo-electric effect. [Punjab 2000, 02, 03, 04]
3. Establish Einstein's photo-electric equation. Use this equation to explain the laws of photo-electric emission. [Himachal 03; Punjab 02, 04]
4. Draw properly labelled graphs to show the following concerning photo-electric emission :
- Variation of photo-electric current with the intensity of incident radiation.
 - Variation of photo-electric current with accelerating and stopping potential.
 - Variation of stopping potential with frequency of the incident radiation. From the graph, how the following can be determined.
 - Planck's constant.
 - The work function of the material. [ISCE 93]
5. Obtain Einstein's photo-electric equation. Explain how it enables us to understand the
- independence of maximum energy of the emitted photo-electrons from the intensity of incident light
 - linear dependence of the maximum energy of the emitted electrons on the frequency of the incident radiation, and
 - the existence of a threshold frequency for a given photoemitter. [CBSE OD 04C]
6. (a) Write three observed features of photoelectric effect which cannot be explained by wave theory of light.
Explain how Einstein's photoelectric equation is used to describe these features satisfactorily.
- (b) Figure 11.38 shows a plot of stopping potential (V_0) with frequency (ν) of incident radiation for two photosensitive materials M_1 and M_2 . Explain

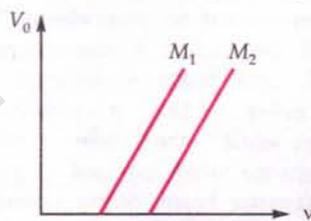


Fig. 11.38

- why the slope of both the lines is same.
- for which material emitted electrons have greater kinetic energy for the same frequency of incident radiation. [CBSE OD 15C]

7. (a) A deuteron and an alpha particle are accelerated with the same accelerating potential. Which one of the two has : (i) greater value of de-Broglie wavelength, associated with it, and (ii) less kinetic energy? Explain.
- (b) A proton and a deuteron are accelerated through the same accelerating potential. Which one of the two has : (i) greater value of de-Broglie wavelength associated with it, and (ii) less momentum? Give reasons to justify your answer. [CBSE D 14; OD 15C]

8. Derive the expression for the de Broglie wavelength of an electron moving under a potential difference of V volt.

Describe Davisson and Germer experiment to establish the wave nature of electrons. Draw a labelled diagram of the apparatus used. [CBSE OD 03]

Answers

1. Refer to point 3 of Glimpses on page 11.61.
2. Refer answer to Q. 4 on page 11.3.
3. Refer to solution of Problem 3 on page 11.28.
4. See Figs. 11.4, 11.5 and 11.7 on page 11.4.
5. Refer to solution of Problem 3 on page 11.28.
6. (a) Refer to the solution of Problem 1(b) and Problem 3 on page 11.28.
(b) Refer to the solution of Problem 11 on page 11.31.
7. (a) (i) de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mqV}}$

For same V , $\lambda \propto \frac{1}{\sqrt{mq}}$

$$\frac{\lambda_d}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_d q_d}} = \sqrt{\frac{4m_p \times 2e}{2m_p \times e}} = 2 \Rightarrow \lambda_d > \lambda_\alpha.$$

(ii) Kinetic energy, $K = qV$

For same V , $K \propto q$

$$\therefore \frac{K_d}{K_\alpha} = \frac{q_d}{q_\alpha} = \frac{e}{2e} = \frac{1}{2} \Rightarrow K_d < K_\alpha$$

$$(b) (i) \frac{\lambda_p}{\lambda_d} = \sqrt{\frac{m_d q_d}{m_p q_p}} = \sqrt{\frac{2m_p e}{m_p e}} = \sqrt{2}$$

Hence, $\lambda_p > \lambda_d$ for the same accelerating potential.

(ii) Momentum, $p = \frac{h}{\lambda}$

As $\lambda_p > \lambda_d \Rightarrow p_p < p_d$

That is the momentum of the proton will be less than that of the deuteron.

8. Refer answer to Q. 15 on page 11.20 and Q. 16 on page 11.20.

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. In an experiment on photoelectric effect, Sumit observed that when a monochromatic yellow coloured light beam is incident on a given photosensitive surface, photoelectrons are not ejected, while the same surface gives photoelectrons when exposed to green coloured monochromatic beam. Sumit consulted his text books but could not understand the reason behind it. He told his problem to his Physics teacher who explained that yellow light has a frequency below the threshold frequency of the given photosensitive surface, so no photoemission occurs with the yellow light.
 - (a) What are the values being displayed by Sumit?
 - (b) What will happen if the same photo-sensitive surface is exposed to (i) violet light and (ii) red light? Give reason.
2. A function was arranged in the school auditorium. The auditorium had the capacity of 500 students. When entry started, students entered in groups and

counting became a great problem. Then, science students took the responsibility at the gate. All the students entered the hall one by one. This helped them to maintain discipline and counting became easy with the help of a device used by these students.

- (a) What values are displayed by science students?
 - (b) Name the device which is based on application of photoelectric effect.
3. Mrs. Sharma's family was fast asleep during night. They had no clue that their living room has caught fire due to a short circuit. Suddenly they heard sound of an alarm and woke up. They were surprised to see that the sound was coming from the model of fire alarm prepared by their son. They were all happy that a small science model had saved their life.
 - (a) Give the values displayed by the parents and the son.
 - (b) Name the device used in the model.

Answers

1. (a) Power of observation, determination and critical thinking.
(b) (i) Violet light ejects photoelectrons because its frequency is greater than that of green light.
(ii) Red light will not eject photoelectrons because its frequency is less than that of yellow light.
2. (a) Knowledge and sense of responsibility.
(b) Photo-cell connected to a digital counter.
3. (a) Knowledge and scientific thinking.
(b) A photo-cell connected to an electric bell. When the fire breaks out, light radiations fall on the photo-cell. Photo-electric current begins to flow and electric bell starts operating as warning signal.

COMPETITION SECTION

Dual Nature of Radiation and Matter

GLIMPSES

1. **Electron.** It is an elementary particle having a negative charge of 1.6×10^{-19} C and mass 9.1×10^{-31} kg.

2. **Work function.** The minimum amount of energy required by an electron to just escape from the metal surface is known as work function of the metal. It is denoted by W_0 .

3. **Electron emission.** The phenomenon of emission of electrons from a metal surface is called electron emission. It is of the following types :

(i) **Thermionic emission.** Here electrons are emitted from the metal surface with the help of thermal energy.

(ii) **Field or cold cathode emission.** Electrons are emitted from a metal surface by subjecting it to a very high electric field.

(iii) **Photoelectric emission.** Electrons are emitted from a metal surface with the help of suitable electromagnetic radiations.

(iv) **Secondary emission.** Electrons are ejected from a metal surface by striking fast moving electrons over it.

4. **Kinetic energy gained by an electron.** When an electron is accelerated from rest through a potential difference of V volts, the gain in its kinetic energy is

$$eV = \frac{1}{2}mv^2$$

5. **Electron volt (eV).** It is the kinetic energy gained by an electron when it is accelerated through a potential difference of 1 volt.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}, \quad 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$$

The work function of a metal is generally measured in electron volt (eV).

6. **Photons.** According to Planck's quantum theory of radiation, an electromagnetic wave travels in the form of discrete packets of energy called quanta. One quantum of light radiation is called a photon. The main features of photons are as follows :

(i) A photon travels with the speed of light.

(ii) The frequency of a photon does not change as it travels from one medium to another.

(iii) The speed of a photon changes as it travels through different media due to the change in its wavelength.

(iv) The rest mass of a photon is zero *i.e.*, a photon cannot exist at rest.

(v) Energy of a photon, $E = hv = \frac{hc}{\lambda}$.

(vi) Momentum of a photon, $p = mc = \frac{hv}{c} = \frac{h}{\lambda}$.

(vii) From Einstein's mass-energy relationship, the equivalent mass m of a photon is given by

$$E = mc^2 = hv \quad \text{or} \quad m = \frac{hv}{c^2}$$

7. **Photoelectric effect.** The phenomenon of emission of electrons from a metal surface, when electromagnetic radiations of sufficiently high frequency are incident on it, is called photoelectric effect. The photo (light)-generated electrons are called photoelectrons.

Alkali metals like Li, Na, K, Cs show photoelectric effect with visible light. Metals like Zn, Cd, Mg respond to ultraviolet light.

Photoelectric effect involves the conversion of light energy into electrical energy. It follows the law of conservation of energy. It is an instantaneous process.

8. **Photoelectric current.** The current constituted by photoelectrons is called photoelectric current. Its value depends on :

- (i) the intensity of light,
- (ii) the potential difference applied between the two electrodes, and
- (iii) the nature of the cathode material.

9. **Cut off or stopping potential.** It is the minimum value of the negative potential that must be applied to the anode of photo-cell to make the photoelectric current zero. It is denoted by V_0 . Its value depends on : (i) the frequency of incident light, and (ii) the nature of the cathode material. For a given frequency of incident light, it is independent of its intensity. The stopping potential is directly related to the kinetic energy of the emitted electrons.

$$K_{\max} = \frac{1}{2} m v_{\max}^2 = eV_0$$

10. **Threshold frequency.** The minimum value of the frequency of incident radiation below which the photoelectric emission stops altogether is called threshold frequency. It is denoted by ν_0 and is a characteristic of the metal.

11. **Laws of photoelectric effect.** (i) For a given metal and a radiation of fixed frequency, the rate of emission of photoelectrons is proportional to the intensity of incident radiation. (ii) For every metal, there is a certain minimum frequency below which no photoelectrons are emitted, howsoever high is the intensity of incident radiation. This frequency is called *threshold frequency*. (iii) For the radiation of frequency higher than the threshold frequency, the maximum kinetic energy of the photoelectrons is directly proportional to the frequency of incident radiation and is independent of the intensity of incident radiation. (iv) The photoelectric emission is an *instantaneous process*.

12. **Failure of wave theory to explain photoelectric effect.** The classical wave theory of radiation could not explain the main features of photoelectric effect. Its picture of continuous absorption of energy from radiation could not explain (i) the independence of K_{\max} on intensity, (ii) the existence of threshold frequency ν_0 and (iii) the instantaneous nature of the phenomenon.

13. **Einstein's theory of photoelectric effect.** Einstein explained photoelectric effect with the help of Planck's quantum theory. When a radiation of frequency ν is incident on a metal surface, it is absorbed in the form of discrete packets of energy called quanta or photons. A part of energy $h\nu$ of a photon is used in removing the electron from the metal surface and remaining energy is used in giving kinetic energy to the photoelectron. Einstein's photoelectric equation is

$$K_{\max} = \frac{1}{2} m v_{\max}^2 = eV_0 = h\nu - W_0 = h(\nu - \nu_0)$$

where W_0 is the work function of the metal and ν_0 is the threshold frequency.

All the experimental observations can be explained on the basis of Einstein's photoelectric equation.

14. **Compton scattering.** It is the phenomenon of increase in the wavelength of X-ray photons which occurs when these radiations are scattered on striking an electron. Then difference in the wavelength of scattered and incident photons is called *Compton shift*, which is given by

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos \phi)$$

where ϕ is the angle of scattering of the X-ray photon and m_0 is the rest mass of the electron.

15. **Photocell.** It is an arrangement which converts light energy into electric energy. It works on the principle of photoelectric effect. It is used in cinematography for the reproduction of sound.

Photo-cells are used to operate various control systems and in light measuring devices.

16. **Dual nature of radiation.** Light has dual nature. It manifests itself as a wave in diffraction, interference, polarisation, etc., while it shows particle nature in photoelectric effect, Compton scattering, etc.

17. **Dual nature of matter.** As there is complete equivalence between matter (mass) and radiation (energy) and the principle of symmetry is always obeyed, de Broglie suggested that moving particles like protons, neutrons, electrons, etc. should be associated with waves known as *de Broglie waves* and their wavelength is called *de Broglie wavelength*. The de Broglie

wavelength of a particle of mass m moving with velocity v is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where h is the Planck's constant. The de Broglie wavelength is independent of the charge and nature of the material particles. It has significantly measurable values for sub-atomic particles like electrons, protons, etc., due to their small masses. For macroscopic objects of everyday life, the de-Broglie wavelength is extremely small, quite beyond measurement.

18. **Davisson and Germer experiment.** This electron diffraction experiment has verified and confirmed the wave-nature of electrons.

19. **de Broglie wavelength of an electron.** The wavelength associated with an electron beam accelerated through a potential difference of V volts is given by

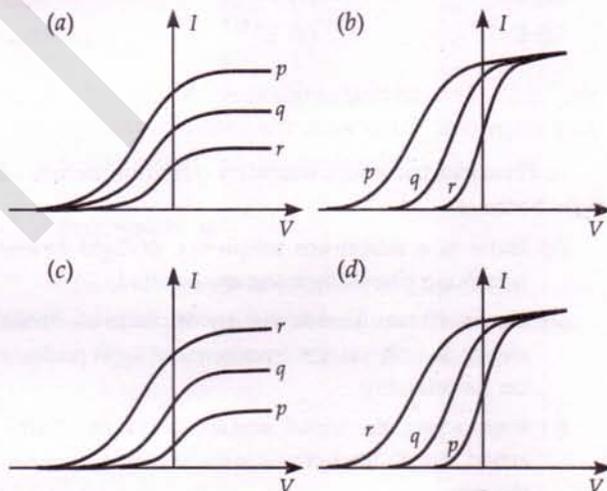
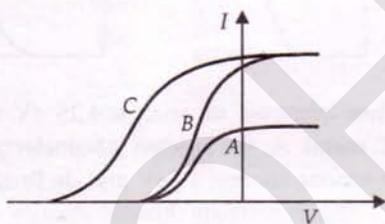
$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{1.227}{\sqrt{V}} \text{ nm}$$

20. **Electron microscope.** It is a device that exploits the wave-nature of electrons to provide high resolving power. It is used to investigate the structural details of bacteria, viruses, etc. It has proved to be a powerful tool of investigation for research in science, technology, metallurgy, industry, medicine, etc.

JEE Advance

Multiple Choice Questions with one correct answer

1. In a photoelectric experiment anode potential is plotted against plate current.



- (a) A and B will have same intensities while B and C will have different frequencies
 (b) B and C will have different intensities while A and B will have different frequencies
 (c) A and B will have different intensities while B and C will have equal frequencies
 (d) B and C will have equal intensities while A and B will have same frequencies. [IIT 2004]

2. Photoelectric effect experiments are performed using three different metal plates p , q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I - V graph for the experiment is [IIT 2009]

3. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential, in volt, is

- (a) 2 (b) 4
 (c) 6 (d) 10

[IIT 1997]

4. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2 , respectively. If the ratio $u_1 : u_2 = 2 : 1$ and $hc = 1240$ eV nm, the work function of the metal is nearly

- (a) 3.7 eV (b) 3.2 eV
 (c) 2.8 eV (d) 2.5 eV

[JEE Adv. 14]